DAMPING SEISMIC WAVES USING ELECTRO-RHEOLOGICAL DAMPERS

A Thesis

by

ANGEL ROBERTO GOMEZ CONSTANTE

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Chair of Committee,	Kumbakonam Rajagopal
Co-Chair of Committee,	Anastasia Muliana
Committee Member,	Jean-Luc Guermond
Head of Department,	Andreas Polycarpou

May 2016

Major Subject: Mechanical Engineering

ABSTRACT

The recent seismic scale that the world has been experiencing, with earthquakes of uncommon intensity has triggered the alarms of the world scientific community due to the devastating aftermaths in terms of infrastructure damage and destruction as well as loss of human lives. It is necessary to develop special mechanical systems that can attenuate such catastrophic effects. One relatively new method for vibration control is the utilization of dampers that use a so called electro-rheological viscous fluid which can experience a substantial change in its viscosity as a controlled electric field runs through it. This methodology is known as a semi-active control system, which, instead of introducing active control forces or passive vibration absorbers, uses a variable rate of damping or stiffness.

The main objective of this work is to analyze this non-linear system response using numerical approximations, in which the damping force depends on the electric field, which at the same time, could depend on the relative velocity of the system. As a first task, the damping force is defined for this semi-active vibration control system. The electric field stands out as the damping modifier, therefore, it is the main input variable of the analysis. Later, a group of cases of study are defined in order to analyze the effect of the variation of the electric field on the damping force and system response. The cases of study where the damping force is analyzed are 1) electric field is zero, 2) electric field is constant and equal to the maximum, and, 3) electric field is a function of the relative velocity of the

system ($E = E(v)$). For these cases of study, the reduce of system response amplitude is significant, which proves the benefits of the method.

DEDICATION

To my father, mother, and my siblings, for all their unconditional love and support.

To my grandfather, Feliciano, for this was the biggest dream of his whole life.

ACKNOWLEDGEMENTS

I would like to start by expressing my eternal gratitude to Prof. K.R. Rajagopal for helping me find the right path that I was looking for in academia and science. This was a mere dream when it all started, and his valuable guidance made this an enriching life time opportunity. Moreover, having attended to his lectures in Continuum Mechanics was the epilogue of this experience and which finally convinced me to continue and pursue a PhD. His extensive knowledge in most scientific fields was always inspiring and woke up inside me the researcher spirit.

I would also like to thank to my committee members: Dr. Anastasia Muliana, for her help and because her lectures on Advanced Solid Mechanics also pushed me into getting involved in my future PhD studies; and Dr. Jean-Luc Guermond for his lectures from both Methods of Applied Mathematics I, and Methods and Applications of Partial Differential Equations which I used in the development of this thesis.

Thanks also go to the Secretaría de Educación Superior, Ciencia, Tecnología e Innovación, and to the Ecuadorian Government represented by Dr. Rafael Correa Delgado, which provided the means for the present work, and for their vision of a new Ecuador in which human talent is going to be its principal source of development.

Finally, thanks to my parents because this is the result of their hard work and all the patience they had as I was growing up.

NOMENCLATURE

FEMA	Federal Emergency Management Agency
F	Force
F_d	Damping force
F_s	Spring force
m, m_1, m_2	Mass
k, k_1, k_2	Stiffness
η, c, c_0, c_1	Damping coefficient
ζ_0, ζ_1	Damping ratio
x, y, z	Displacement
$v, \frac{dx}{dt}, \dot{x}, \dot{y}, \dot{z}$	Velocity
$a, \ddot{x}, \ddot{y}, \ddot{z}$	Acceleration
g	Gravity
P-waves	Primary or compressional waves
S-waves	Secondary or shear waves
ER	Electro-Rheological
MR	Magneto-Rheological
E	Electric field
DAE	Differential algebraic equations
T	Kinetic energy
V	Potential energy

dW	Work variation
$\vec{r}$	Position vector
Q_z	External force
L	Lagrangian
T_{ij}, σ_0	Stress
μ	Viscosity
γ	Shearing
ω, ω_n	Frequency
f_q, f_n	Frequency
$t, \Delta t, t_f, d$	Time
f, p, h	Functions

TABLE OF CONTENTS

Page

ABSTRACT ii

DEDICATION iv

ACKNOWLEDGEMENTS v

NOMENCLATURE vi

TABLE OF CONTENTS viii

LIST OF FIGURES x

CHAPTER I INTRODUCTION AND LITERATURE REVIEW 1

Earthquake Effects on Buildings 1
Inertial Forces and Accelerations 1
Duration, Velocity, and Displacement 2
Ground Amplification 2
Frequency and Resonance 3
Natural Frequency 3
Ground Motion, and Building Resonance 4
Site Response Spectrum 4
Damping 5
Dynamic Amplification 6
Higher Forces and Uncalculated Resistance 6
Ductility 7
Strength, Stiffness, Force Distribution, and Stress Concentration 7
Strength and Stiffness 7
Force Distribution and Stress Concentration 8
Torsional Forces 9
Nonstructural Components 10
Construction Quality 11
Vibration Control Systems 11
Passive Vibration Control Method 11
Active Vibration Control Method 12

Semi-Active Vibration Control Method 13
Electro-Rheological Fluids 14

CHAPTER II PROBLEM DESCRIPTION AND SOLUTION PROCESS 15

Problem Description 15
Derivation of the Equation of Motion 16
Constitutive Relations Development 18
Spring Force Definition, Energy Storage 18
Damping Force Definition, Energy Dissipation 19
Electrical Field Control Function Definition 25
External Force Definition 26
Numerical Solution for Differential Algebraic Equations 29
Electric Field Discretization 31
Solution Algorithm Definition 33

CHAPTER III PARAMETER DEFINITION, RESULTS AND DISCUSSION, AND CONCLUSIONS 35

Parameter Definition 35
Results and Discussion 42
Displacement Response 44
Velocity Response 47
Acceleration Response 50
Spring Force Response 53
Damping Force Response 56
Damping Force vs. Velocity 58
Conclusions 60

REFERENCES 62

LIST OF FIGURES

Page

Figure 1. Natural frequencies determined by height. 3

Figure 2. Buildings from 6 to 20 stories are more like to resonate during an earthquake because their frequencies are very similar to most seismic events. 4

Figure 3. Response spectrum for acceleration, velocity and displacement. 5

Figure 4. Response spectrum for various values of damping. 6

Figure 5. Difference in the stiffness by changing of orientation 8

Figure 6. Short column condition. 9

Figure 7. Torsional forces. 10

Figure 8. Nonstructural systems. 10

Figure 9. Passive vibration controller scheme. 12

Figure 10. Active vibration controller scheme. 13

Figure 11. Semi-active vibration controller scheme. 13

Figure 12. Basic system scheme. 15

Figure 13. Damping wall scheme. 20

Figure 14. Cylindrical dashpot scheme. 20

Figure 15. Shear stress – shear rate relation for increasing electric field E. 21

Figure 16. Damping force – velocity relation for increasing electric field E. 22

Figure 17. Velocity – damping force relation for increasing electric field E. 23

Figure 18. Electric field – velocity control function. 26

Figure 19. Harmonic part of the earthquake motion. 27

Figure 20. Bump function. 28

Figure 21. Harmonic function representing the earthquake motion................................29
Figure 22. Discretization of the electric field control function. 32
Figure 23. Earthquake sine-bump forcing function for Case 1 37
Figure 24. Earthquake acceleration for Case 1...37
Figure 25. Earthquake sine-bump forcing function for Case 2 38
Figure 26. Earthquake acceleration for Case 2...39
Figure 27. Sine forcing function for Case 3 40
Figure 28. Acceleration for Case 3...40
Figure 29. Earthquake sine-bump forcing function for Case 4 41
Figure 30. Earthquake acceleration for Case 4...42
Figure 31. Algorithm implementation...43
Figure 32. Case 1: Displacement response...44
Figure 33. Case 2: Displacement response...45
Figure 34. Case 3: Displacement response...45
Figure 35. Case 4: Displacement response...46
Figure 36. Case 1: Velocity response..47
Figure 37. Case 2: Velocity response..48
Figure 38. Case 3: Velocity response..48
Figure 39. Case 4: Velocity response..49
Figure 40. Case 1: Acceleration response. 50
Figure 41. Case 2: Acceleration response. 50
Figure 42. Case 3: Acceleration response. 51
Figure 43. Case 4: Acceleration response. 52
Figure 44. Case 1: Spring force response...53

Figure 45. Case 2: Spring force response. 54

Figure 46. Case 3: Spring force response. 54

Figure 47. Case 4: Spring force response. 55

Figure 48. Case 1: Damping force response. 56

Figure 49. Case 2: Damping force response. 56

Figure 50. Case 3: Damping force response. 57

Figure 51. Case 4: Damping force response. 57

Figure 52. Case 1: Damping force vs velocity. 58

Figure 53. Case 2: Damping force vs velocity. 59

Figure 54. Case 3: Damping force vs velocity. 59

Figure 55. Case 4: Damping force vs velocity. 60

CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

A brief explanation of the factors that affect the performance of structures during a seismic event are explained in the following paragraphs. Additionally, systems and methods for controlling the vibrating response are discussed here.

Earthquake Effects on Buildings

In order to perform an analysis of a possible attenuating solution to building failures due to earthquakes it is important to take a brief look to the set of attributes that affect the building performance in a seismic event. These attributes are for both ground and building structures.

Inertial Forces and Accelerations

According to the Federal Emergency Management Agency (FEMA) [1], among the most representative attributes are inertial forces and accelerations. The explanation of the inertial forces comes from Newton's Second Law of Motion

$$\sum F = m \cdot a \tag{1}$$

The mass of the building plays a significant role in the inertial force that it will experience during an earthquake. That is why light weight buildings, like wood frame structures, perform better during seismic events.

Acceleration of the ground caused by the seismic waves, as a percentage of the gravity acceleration, is of importance. For a moderate seismic event those accelerations may be approximately 0.2g (poorly constructed building begin to suffer substantial damage at 0.1g).

Duration, Velocity, and Displacement

An effective measure of duration is the so-called bracketed duration: the time between the first and the last peak when the acceleration reaches its threshold value of 0.05g. This varies from a few seconds to as long as several minutes.

Velocity is the rate of motion of a seismic wave as it travels through the earth. For P-waves the velocity varies from 3 km/sec to 8 km/sec, while for the slower S-waves it can be from 2 km/sec to 5 km/sec. The motion of the ground is slower, going from 2 cm/sec for a small earthquake to about 60 cm/sec for a bigger event. Hence, even though wave velocity can be significant, the real building motion, in general, is slow with small displacements.

The displacement measure is the distance that the points on the ground move from their reference initial position due to seismic waves. Buildings closer to the epicenter will have bigger displacements than the ones located farther away.

The frequency of the motion of the waves will also affect the ground acceleration, velocity and displacement substantially. High-frequency waves (higher than 10 Hz) will produce higher acceleration but smaller displacement amplitudes, whereas low-frequency waves will have small amplitudes of acceleration but much larger velocities and displacements.

Ground Amplification

Ground amplification depends on the soil nature. Soft soil is the riskiest with amplification factors going from 1.5 to 6, for soil layers of a few feet to about a hundred.

The frequency of the seismic event also influences the ground amplification, with the highest amplification for low-frequency, and barely significant amplification for high-frequency earthquakes.

Frequency and Resonance

Natural Frequency

Natural frequency is the number of cycles that all bodies will make in a second if they are given an initial push. In structural engineering, a first approximation of the natural frequency of a building can be calculated dividing 10 Hz to the number of stories (Figure 1).

After a seismic event a structure suffers a softening effect due to fractures and internal failures in the building, causing its natural frequency to decrease. This softening may produce a resonance in a new earthquake.

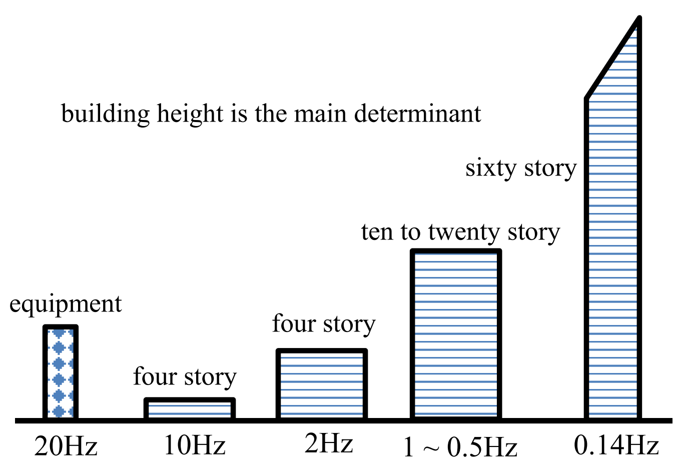

Figure 1. Natural frequencies determined by height. This approximation does not take into account materials, geometric proportions, or structure type.

Ground Motion, and Building Resonance

The ground also has a natural frequency of vibration when an earthquake shakes it. Its natural frequency depends on the soil formation. Soils formed with harder materials have higher natural frequencies. In general, natural frequency of ground goes from 0.5Hz for soft soil to 2.5Hz for hard ground. Adding these values of natural frequencies for the ground, to the values for common natural frequencies for buildings reviewed above, makes a resonance effect very probable for buildings from six to twenty stories (Figure 2).

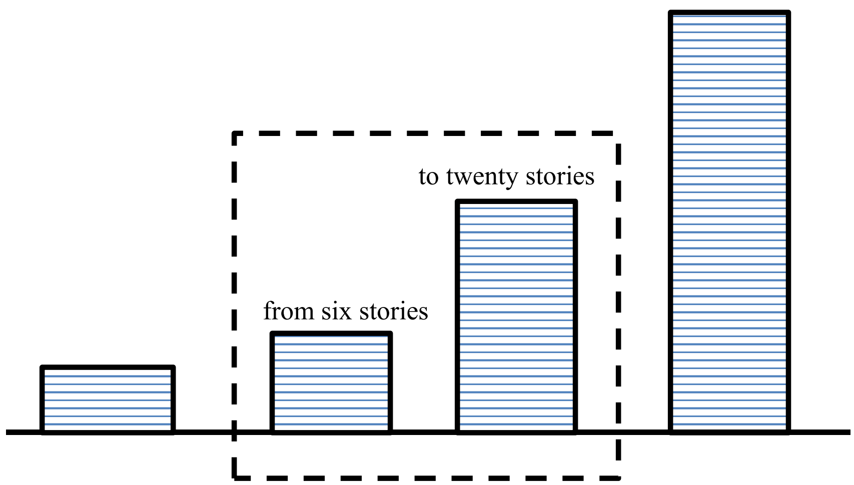

Figure 2. Buildings from 6 to 20 stories are more like to resonate during an earthquake because their frequencies are very similar to most seismic events.

Buildings of 20 stories or more will also experience several modes of vibration that will make them go back and forth like a snake, nevertheless, natural frequency will be always more critical than the higher modes.

Site Response Spectrum

For the same seismic event there can be a wide range of site responses depending on the specific building configuration. An engineering tool was developed for this, the site response spectrum. This tool is a graph (Figure 3) which specifies the maximum responses in acceleration, velocity and displacement as a function of the natural period (inverse of the frequency) of an analyzed structure.

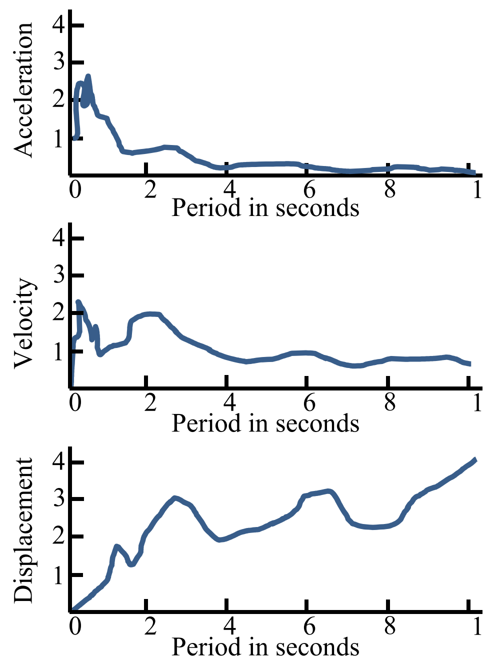

Figure 3. Response spectrum for acceleration, velocity and displacement.

Damping

Damping (Figure 4) is known as the decrease on energy due to internal composition of a structure, and absorption of energy of many kinds.

The measure used to evaluate the damping on a structure is the damping ratio, which is the rate between the damping coefficient of the building and the critical damping.

The critical damping is the least amount of damping that will allow a structure to return to its original position without any oscillation.

Structures have damping ratios from 0.03 to 0.10. Older structures have higher values, while newer modern steel frame structures have smaller ratios.

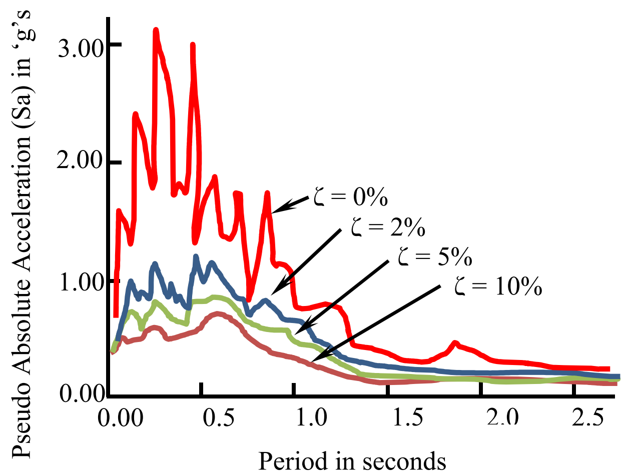

Figure 4. Response spectrum for various values of damping.

Dynamic Amplification

Dynamic amplification is directly related to both natural frequency and the damping properties of the structure. It is the increase in movement over the movement of the ground.

Taking into account a regular design damping ratio value like 0.05 and a natural frequency range of 0.3 Hz to 2 Hz, the dynamic amplification would be about 2.5.

Higher Forces and Uncalculated Resistance

There are forces that affect a structure during an earthquake that are higher than the ones for which the structure was designed. Those forces are expected to cause high damage into the structures, they finally do not.

This is due to the use of safety factors, which allow such structures to carry a reserve resistance which adds to the design strength. That extra strength comes also in some cases from other secondary elements including division walls, or brackets from support installations.

Newer seismically design steel frame structures have an additional source of reserve strength which comes from the ductility of the materials.

Ductility

This property fills up the difference between the actual forces acting during an earthquake and the design capacity of the system.

It is an indicator of the extra strength capacity of a material, particularly, steel, after it experiences plastic deformation and before it breaks.

It allows structures to get larger deformations while still keeping some strength, allowing them stay erect even though they become useless.

Strength, Stiffness, Force Distribution, and Stress Concentration

Strength and Stiffness

The strength is related to the internal force that a material can hold as external forces are applied.

The stiffness, on the other hand, relates to the deflection that a material can bend as a part of a structural member.

Strength and stiffness are closely related. However, for two structural members with very similar material strengths there may be a complete variation in their strength and stiffness by just changing their orientation relative to the loads they carry (Figure 5).

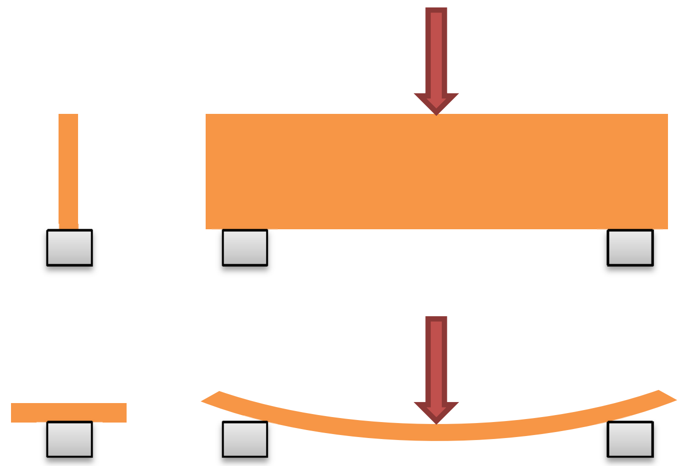

Figure 5. Difference in the stiffness by changing of orientation. The strength is the same in both cases.

Force Distribution and Stress Concentration

Engineering work has determined that each resisting element carries a proportional part of the overall load as a function of its relative stiffness, in other words, the load tends to concentrate at the stiffer structural members.

The element that carries the highest amount of load is the one with the higher stiffness. The only way that two elements can carry the same load is if they have the same stiffness.

The stiffness of a column is approximately proportional to the inverse of the cube of its length, which means that if there are two columns and one has half the length of the other, the shortest one will have 8 times the stiffness of the first one, but it will also carry 8 times more load. This phenomenon is known as short column condition (Figure 6).

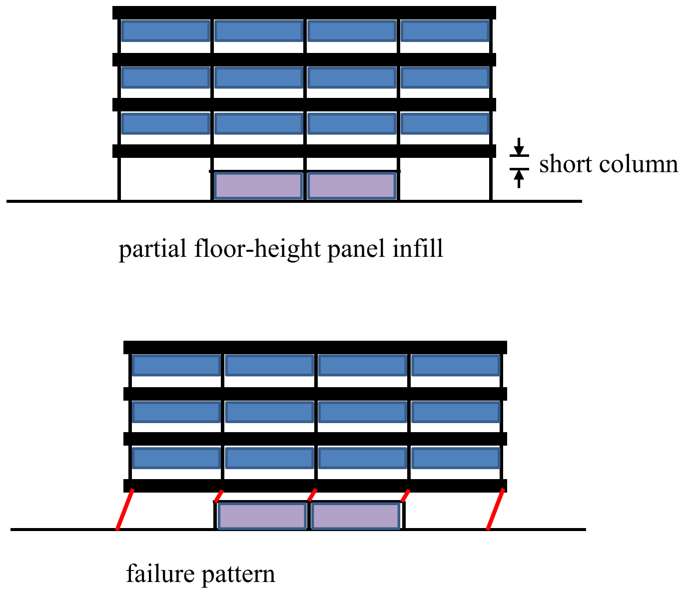

Figure 6. Short column condition.

Torsional Forces

Torsional forces are created by an unbalance in the structural elements (Figure 7) of a building and produces an eccentricity between the center of mass and the center of resistance.

Seismic design attempts to reduce this unbalance as much as possible by making symmetrical structures; nevertheless, seismic codes take this into account and make provision for this given that such torsion cannot be avoided completely.

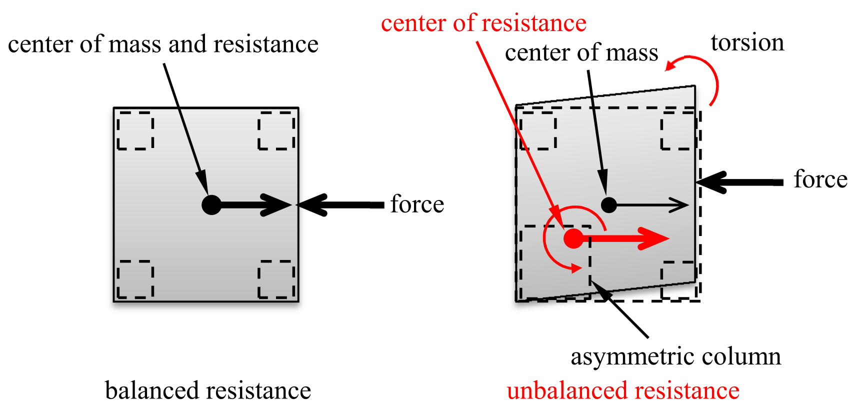

Figure 7. Torsional forces.

Nonstructural Components

Although seismic design does not take nonstructural components into account, sometimes they are cause of failure during an earthquake. Heavy partitions, brackets for piping, heavy equipment, or any heavy structure can either overload an area or can produce the short column effect (Figure 8).

Figure 8. Nonstructural systems.

Construction Quality

There is no mechanical system that can accomplish its design intent if it is not well constructed.

Construction quality not only means that the elements have to be built with high quality standards and that the connections have to be tested, but it also means that the materials that are used should be guaranteed to have the mechanical and physical properties necessary in order to perform the functions for which they are selected.

Vibration Control Systems

Several methods for vibration control have been studied and implemented depending on the field of application.

In order to mitigate vibration response, three methods have been developed: passive, active and semi-active vibration control methods. Each of these methods have pros and cons and all can be applied to the different fields of engineering.

Passive Vibration Control Method

The passive vibration control method (Figure 9) needs a complimentary device that detunes the main system's natural frequencies and thus avoids the possibility of a resonance in the range of work frequencies.

This complimentary device has a mass, and a tunable stiffness to help dissipate energy. It normally has the same natural frequency as the main system to which it is attached.

Good examples of this method are the mechanical vibration absorbers which can be located on the tip of the wings of a plane to avoid resonance and damage to equipment, or under a bridge to diminish the effects of the wind.

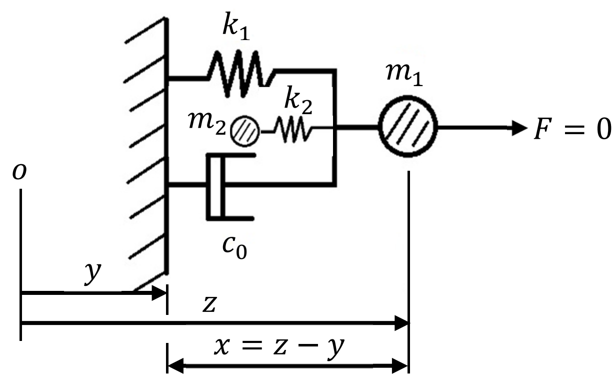

Figure 9. Passive vibration controller scheme.

The main problem with this method is that the devices tend to get worn and detuned with time and usage, so they are required to have maintenance plans to make sure they can keep working in the right range.

Active Vibration Control Method

The active vibration control method (Figure 10) overcomes the main problem of the passive control, the detuning. The active method uses sophisticated devices such as actuators, sensors, valves, and also control algorithms to provide control performance inside a range of operation and with parameters permanent in time, making it less influenced by external disturbance.

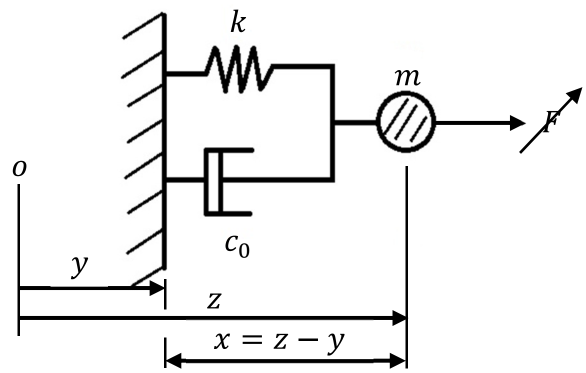

Figure 10. Active vibration controller scheme.

Semi-Active Vibration Control Method

The semi-active vibration control method (Figure 11) combines the passive control low budget equipment and parts with the active sophisticated control.

This method is predicated on the control of the basic parameters like stiffness and damping, rather than introducing external forces.

Electro-Rheological (ER) or Magneto-Rheological (MR) Fluids, which can change their viscosities when they are induced with a small variation of Electric or Magnetic Field, respectively, are the most commonly used materials in this method.

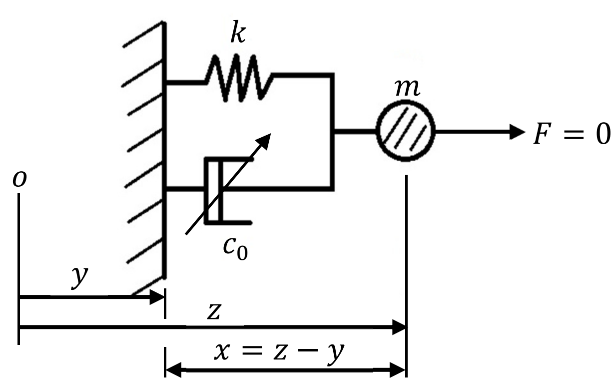

Figure 11. Semi-active vibration controller scheme.

This thesis work focuses on an application of a semi-active control method using an ER fluid in which the electric field E flows based on the velocity of the mass of the system.

Electro-Rheological Fluids

An electro-rheological fluid is a suspension of non-conductive particles in an electrically insulating fluid [2]. When an electric field is applied to this fluid, its viscosity changes in a matter of milliseconds. For example, it can go from liquid consistency to that of a gel, and back.

This property is widely used for some mechanical devices such as dashpots for specific applications where a close control of the damping ratio ζ is needed. Such systems have been previously defined as semi-active. With a minor control system, they adjust the damping in the dashpot according to the conditions working on the system.

CHAPTER II

PROBLEM DESCRIPTION AND SOLUTION PROCESS

In this chapter the unknown elements of the system will be defined and the solution process will be posed completely. In the final lines a full differential algebraic equations (DAE) system will be given, which describes this problem scope, as well as the solution method.

Problem Description

For the basic mass, dashpot, spring system shown in the Figure 12, subjected to external motion y, the differential equation governing the response has to be analyzed with respect to the relative motion x, measured between the absolute motion z of the mass m, and the vibrating surface motion y.

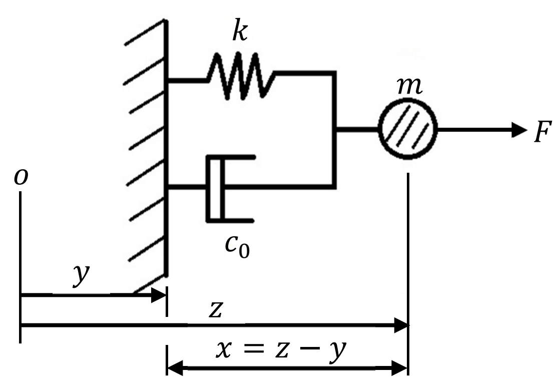

Figure 12. Basic system scheme.

$$x = z - y \tag{2}$$

Also, from Eq. (2), both relative velocity and acceleration are defined as follows,

$$\begin{aligned} \dot{x} &= \dot{z} - \dot{y} \\ \ddot{x} &= \ddot{z} - \ddot{y} \end{aligned} \tag{3}$$

The scope of this work is to analyze the response of the system depicted in Figure 12, when the dashpot uses an ER fluid, in the variable x which represents the relative motion of the mass m. A complete vibrations analysis will be developed to define, in the first place, the equation of motion that describes the physical phenomenon, and finally the required constitutive equations that describe each one of the terms in the said equation of motion.

Additionally, once the damping force with an ER fluid has been defined, we will analyze the response of the system when the electric-field E is an increasing function of the relative velocity v.

Derivation of the Equation of Motion

A quick Lagrangian analysis helps find the equation of motion (EOM) of the proposed system. For such analysis both kinetic and potential energies are required. The system produces kinetic energy from its mass moving at its absolute motion z, thus,

$$T = \frac{1}{2} m\dot{z}^2 \tag{4}$$

As the proposed problem does not take the gravity into consideration, the only contribution to the potential energy comes from the spring, then,

$$V = \frac{1}{2} k(z - y)^2 \tag{5}$$

Where, k is the stiffness of the system. Now, the work performed by the external forces can be calculated as follows:

$$dW = \overrightarrow{F_d} \cdot \overrightarrow{dr} + \vec{F} \cdot \overrightarrow{dr} \tag{6}$$

where $\overrightarrow{F_d}$ is the damping force generated by the dashpot, $\vec{F}$ is the external force applied over the mass m, and $\overrightarrow{dr}$ is the differential of the position vector that locates both forces over the mass m. Then,

$$\overrightarrow{F_d} = -F_d\vec{\imath}$$
$$\vec{F} = F\vec{\imath} \tag{7}$$
$$\vec{r} = z\vec{\imath}$$

Replacing Eq. (7) into Eq. (6) and performing the respective dot products and differentials,

$$dW = -F_d dz + F_d dz = (-F_d + F)dz = Q_z dz$$
$$Q_z = -F_d + F \tag{8}$$

where Q_z is the external force acting on the mass m in the z direction. Thus, the corresponding Lagrangian to this application can be expressed as follows,

$$L = T - V = \frac{1}{2}m\dot{z}^2 - \frac{1}{2}k(z-y)^2 \tag{9}$$

Now, applying the Lagrangian analysis renders,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = Q_z$$
$$m\ddot{z} + k(z-y) = -F_d + F \tag{10}$$

Replacing Eq. (2) and Eq. (3b) into Eq. (10b), the above differential equation has the following final form,

$$m\frac{d^2x}{dt^2} = -F_s - F_d + F - m\frac{d^2y}{dt^2}$$
$$F_s = kx \tag{11}$$

where x is the relative displacement from Eq. (2), F_s is the elastic force due to the linear spring, F_d is the dissipative force produced by the dashpot, F is the external force applied over the mass m, and y is the external ground motion exerted by the earthquake. Additionally, for the external ground motion y, it must be assumed that it will be given by a harmonic function, and also that the external force F is going to be zero for the final analysis.

Constitutive Relations Development

Now, for the specific mechanical properties inherent in both the spring and dashpot: in general, constitutive relations can be developed as a function of the relative displacement (2) and relative velocity (3a) respectively.

$$\begin{gathered} F_s \coloneqq F_s(x) \\ F_d \coloneqq F_d\left(\frac{dx}{dt}\right) \end{gathered} \qquad (12)$$

As a consequence, the constitutive relations in Eq. (12) are functions, either linear or not, of their respective variables. In that sense, by defining the functions for Eq. (12), one can go back and replace them in Eq. (11) to form the differential equation governing the motion, and then one can find its initial value solution through a standard method.

Spring Force Definition, Energy Storage

It is assumed that the spring-like element, for this particular application is going to be a linear spring. Hence, it takes the standard linear form as follows,

$$F_s \coloneqq F_s(x) \coloneqq kx \qquad (13)$$

Damping Force Definition, Energy Dissipation

Now, the scope of the problem is that the damping coefficient has to be a function of the electric field E which afterwards will also be controlled as a function of the relative velocity $\dot{x} = \frac{dx}{dt}$ Eq. (3a). This way, the system will increase its damping force as the seismic excitation increases the electrical field as a function of its velocity.

$$F_d \coloneqq F_d\left(\frac{dx}{dt}, E\right) \tag{14}$$

Then it is understood from Eq. (14) that F_d is defined as a non-linear function of $\frac{dx}{dt}$ and E.

Additionally, it is also assumed that the Electro-Rheological fluid in the dashpot will behave as a Newtonian fluid as long as no electric field E is applied. It will also behave like a "Bingham fluid" after the electric field E is applied, and it will show an initial yield force at the instant when the dashpot starts moving.

Modeling a Damper Using a ER Fluid

For seismic applications two devices are mainly used, damping walls and cylindrical dashpots (Figures 13 and 14).

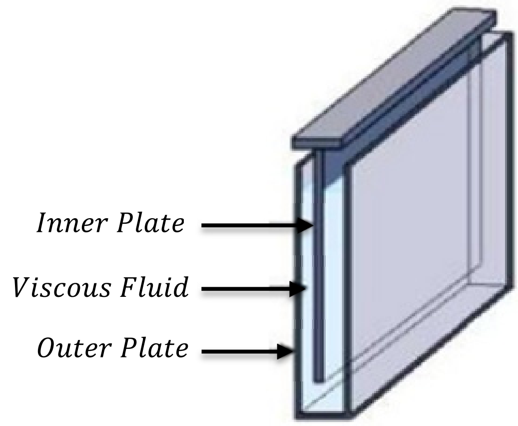

Figure 13. Damping wall scheme.

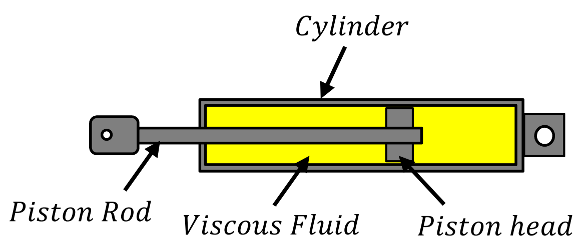

Figure 14. Cylindrical dashpot scheme.

Both systems can be approximated using an analysis of flow in between two parallel plates, followed by analysis using the theory developed by Rajagopal [2] in his work of "Flow of electro-rheological materials" where he defines a relation between the shear stress T_{12}, the electric field E, and the shear-rate γ through a differential equation that helps find the velocity field $v(x_2)$, where x_2 is the transverse direction. In that work, the experimental evidence by Prof. Filisko suggests the following relation,

$$T_{12} = \begin{cases} \sigma_0(E) + \mu(E)\gamma, & \gamma > 0 \\ 0, & \gamma = 0 \\ -\sigma_0(E) + \mu(E)\gamma, & \gamma < 0 \end{cases} \quad (15)$$

where $\mu(E)$ represents the shear viscosity function which depends on the electric field E, $\sigma_0(E)$ represents the yield stress which depends on the electric field E, and γ is the shear rate.

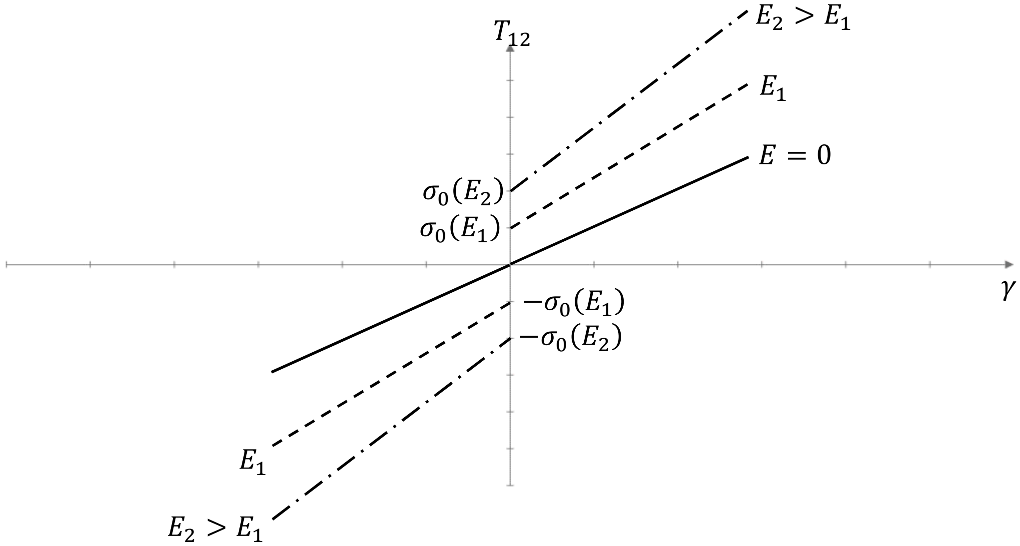

Figure 15. Shear stress – Shear rate relation for increasing electric field E.

It is important to note from Figure 15 that when $E = 0$, the shear response is that of the classical linearly viscous fluid, whereas when $E \neq 0$, the response is similar to that of a Bingham fluid, with yield stress $\sigma_0(E)$ and viscosity $\mu(E)$.

Thus, from Eq. (15) and Figure 15 the damping force for a dashpot with parallel plates can be analogically analyzed as in Figure 16.

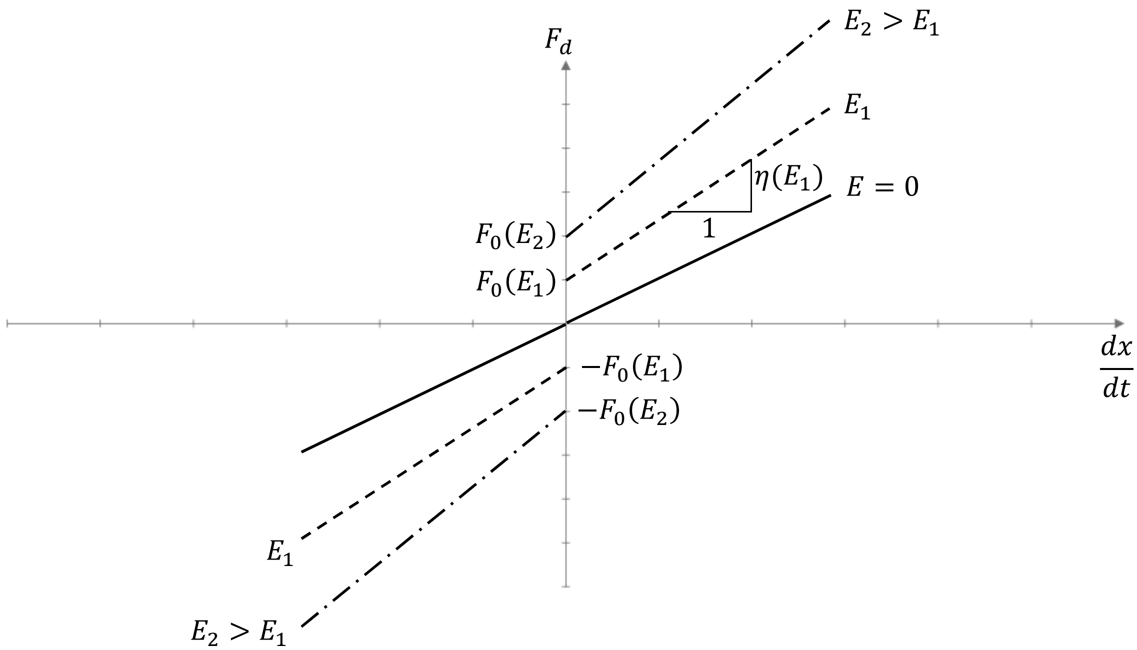

Figure 16. Damping force – Velocity relation for increasing electric field E.

Unlike the standard constitutive equation defined in Eq. (12b), were F_d is a given function of the relative velocity $\frac{dx}{dt}$, this constitutive equation is going to be in the form $\frac{dx}{dt} = f(F_d)$, which is not invertible. This means that F_d cannot be substituted into the differential equation (11), thus non-standard methods to find a solution are needed.

Then, the implicit constitutive relation $\frac{dx}{dt} = f(F_d)$ can be depicted as seen in Figure 17.

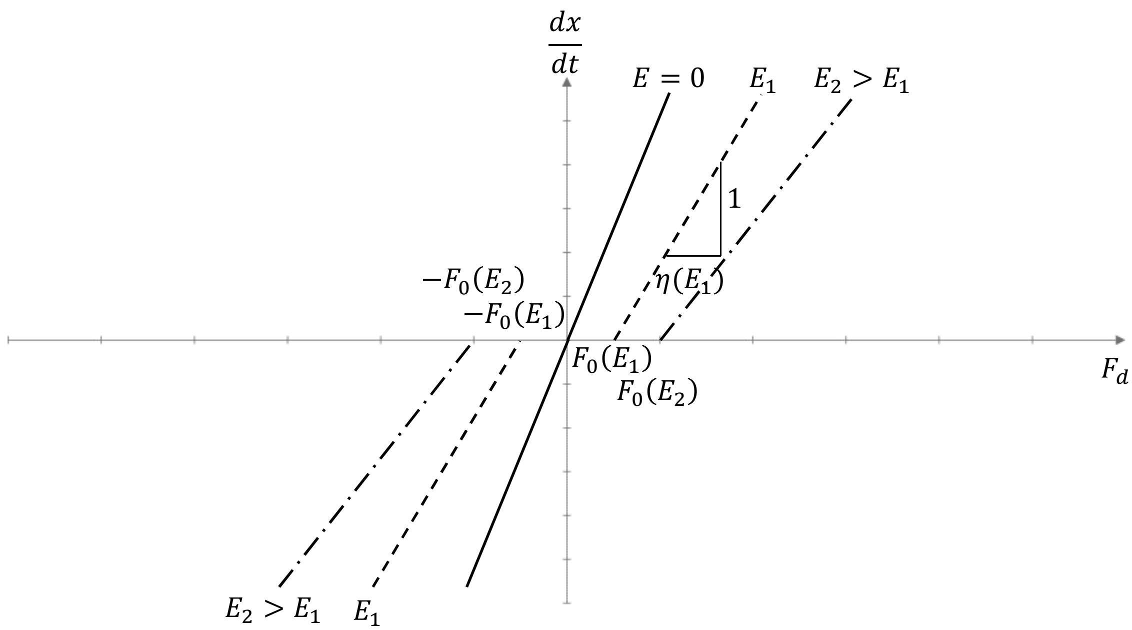

Figure 17. Velocity – damping force relation for increasing electric field E.

From Figure 17, in order to provide a relationship for a dashpot containing an electro-rheological fluid, a damping coefficient $\eta(E)$ and a yield force $F_0(E)$, both increasing monotonically with the electric field E, are needed. Hence, following Darbha's [3] guidelines from the work on "Vibrations of lumped parameters systems governed by differential-algebraic equations," the relative velocity $\frac{dx}{dt}$ will have the following representation,

$$v(t) = \begin{cases} 0, & |F_d(t)| \leq F_0(E), \\ \dfrac{1}{\eta(E)}\left(F_d(t) - \mathrm{sgn}[F_d(t)] \cdot F_0(E)\right), & |F_d(t)| > F_0(E), \end{cases} \qquad (16)$$

$$v(t) = \frac{dx}{dt}$$

where $\eta(E)$ is the damping coefficient, $\frac{1}{\eta(E)}$ is the slope, and $F_{d,ER}(E)$ is the threshold force, and where both $\eta(E)$ and $F_0(E)$ depend on the electric field E applied to the fluid. Additionally, from Figure 17 it is clear that for $|F_d(t)| > F_0(E)$,

$$\text{sgn}\left[\frac{dx}{dt}\right] = \text{sgn}[F_d(t)] \tag{17}$$

It is now necessary to define monotonic increasing functions for both $\eta(E)$ and $F_0(E)$ according to their physical meaning. For the damping coefficient $\eta(E)$ the following equation is given,

$$\eta(E) = c_0 + c_1(E) \tag{18}$$

where c_0 represents the initial damping coefficient of the fluid, and $c_1(E)$ is the increment in damping coefficient as a function of the electrical field E. Now, using the definition of damping ratio $\zeta = \frac{c}{2\sqrt{km}}$ in Eq. (18) renders

$$\eta(E) = 2 \cdot \sqrt{km} \cdot \left(\zeta_0 + \zeta_1(E)\right) \tag{19}$$

where ζ_0 represents the initial damping ratio of the fluid, and $\zeta_1(E)$ is the increment in damping ratio as a function of the electrical field E. Next, an expression for $\zeta_1(E)$ can be defined as,

$$\begin{aligned} \zeta_1(E) &= s_1 E, \\ s_1 &= \frac{\zeta_{1max}}{E_{max}} \end{aligned} \tag{20}$$

where ζ_{1max} and E_{max} are positive constants chosen arbitrarily, and E represents the electric field. Finally, the yield electro-rheological force $F_0(E)$ is defined as follows:

$$\begin{aligned} F_0(E) &= s_2 E, \\ s_2 &= \frac{F_{0max}}{E_{max}} \end{aligned} \tag{21}$$

where $F_{0_{max}}$ and E_{max} are positive constants chosen arbitrarily, and E represents the electric field. Thus, $\zeta_1(E)$ and $F_0(E)$ are effectively defined as functions that increase monotonically with E as stated before.

This way using Eq. (16) to (21) the damping force is finally defined as,

$$v(t) = \begin{cases} 0, & |F_d(t)| \le s_2 E, \\ \dfrac{F_d(t) - \mathrm{sgn}[F_d(t)]\, s_2 E}{2\sqrt{km}(\zeta_0 + s_1 E)}, & |F_d(t)| > s_2 E, \end{cases}$$
$$s_1 = \frac{\zeta_{1_{max}}}{E_{max}} \qquad (22)$$
$$s_2 = \frac{F_{0_{max}}}{E_{max}}$$

Electrical Field Control Function Definition

Additionally, given the definition in Eq. (22) of the relative velocity as a function of the damping force, the following control function for the electric field E as a function of the relative velocity v is proposed to be

$$E(v) = \begin{cases} s_3 v, & |v| \le v_{lim}, \\ \mathrm{sgn}(v)\, E_{lim}, & |v| > v_{lim}, \end{cases}$$
$$s_3 = \frac{E_{lim}}{v_{lim}} \qquad (23)$$

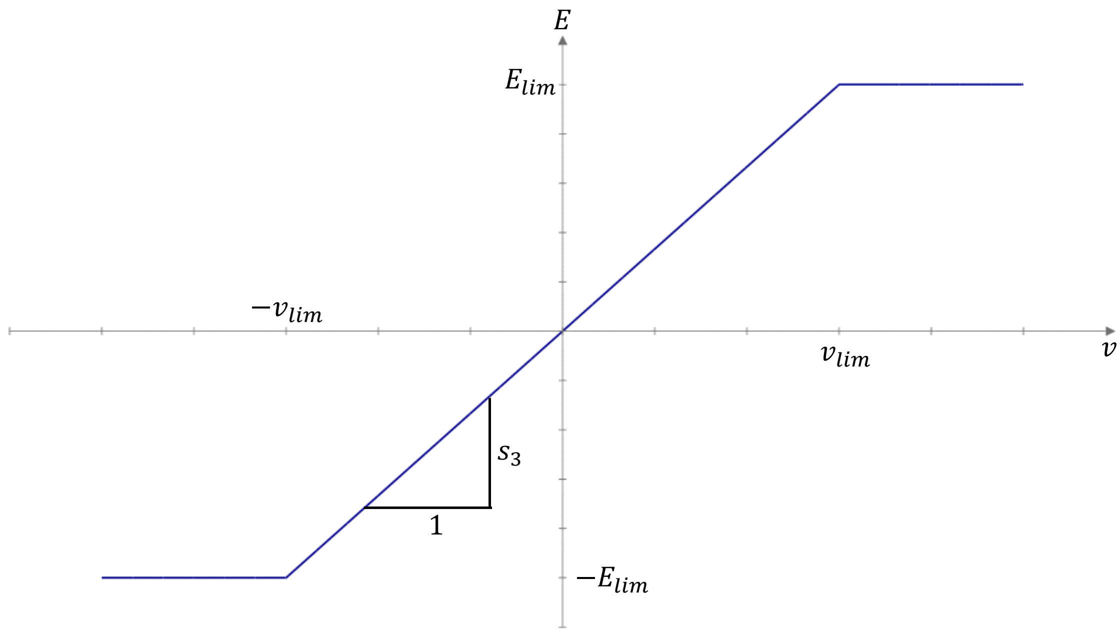

Figure 18. Electric field – velocity control function.

From Figure 18, this function will provide an electrical field E ramp into a given interval $|v| \leq v_{lim}$, such that the response to a sudden seismic event will be under smooth control as the relative velocity ($\dot{x} = v$) increases.

This control function will be used as a third case of study and its results will be compared against those of both electric field at zero and at maximum.

External Force Definition

Now, remembering the equation of motion of the system Eq. (11) there are two forcing expressions yet to be defined, the external force F, and the earthquake motion y. From the scope of the problem, for a semi-active control method, the external force F is equal to zero, thus, the only external influence will be given by the motion y.

As stated above, the motion generated by an earthquake should be represented by a harmonic function. In order to define a consistent harmonic seismic even, the product of two function is used. The first one is a sine function (Figure 19) which carries the displacement amplitude y_0 and the frequency ω of the ground motion.

$$y_1(t) = y_0 \cdot \sin(\omega t), \tag{24}$$

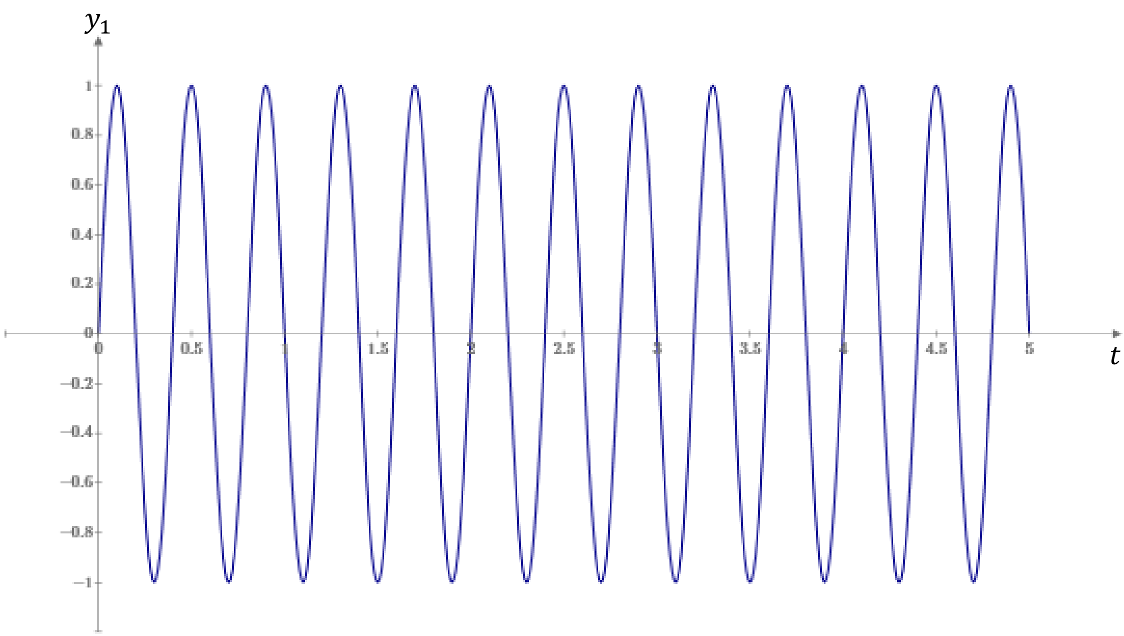

Figure 19. Harmonic part of the earthquake motion.

The second one is a bump function (Figure 20) which helps simulate a smooth transition from zero to a maximum amplitude. Such function has to be infinitely differentiable and with compact support. It is defined as follows,

$$f(t) = \begin{cases} e^{-\frac{1}{t}} & t > 0 \\ 0 & t \leq 0 \end{cases}$$
$$p(t) = \frac{f(t)}{f(t) + f(1-t)} \tag{25}$$

$$h(t) = p\left(t + \frac{d}{2}\right) \cdot p\left(t - \frac{d}{2}\right)$$

$$y_2(t) = h\left(t - \frac{t_f}{2}\right)$$

where t_f is the time frame of the analysis, and d is the domain where $y_2(t) \neq 0$.

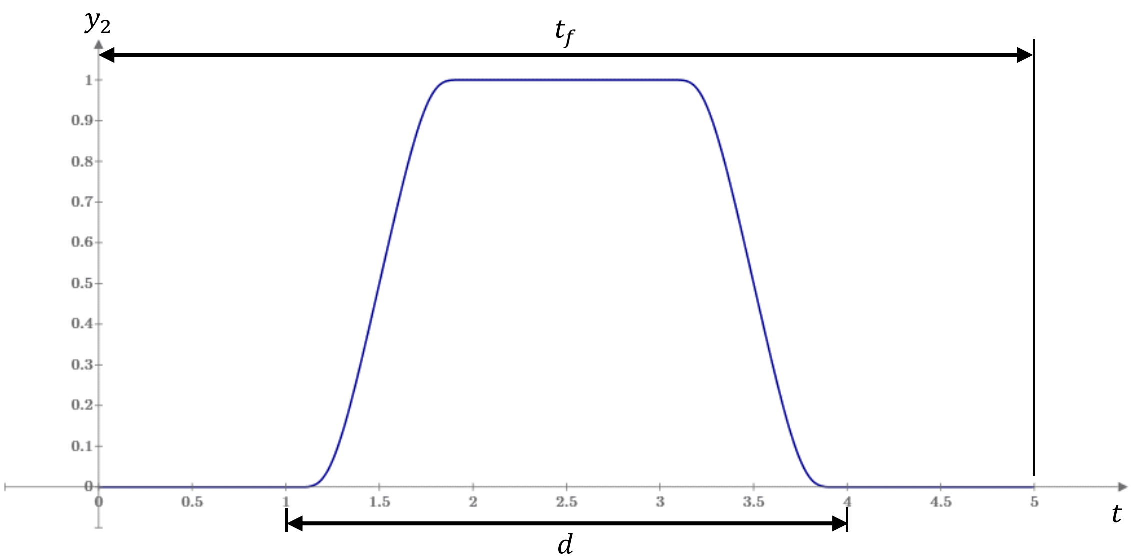

Figure 20. Bump function.

Thus, the final earthquake function (Figure 21) is as follows,

$$y(t) = y_1(t) \cdot y_2(t),$$
$$y(t) = y_0 \cdot h\left(t - \frac{t_f}{2}\right) \cdot \sin(\omega t) \qquad (26)$$

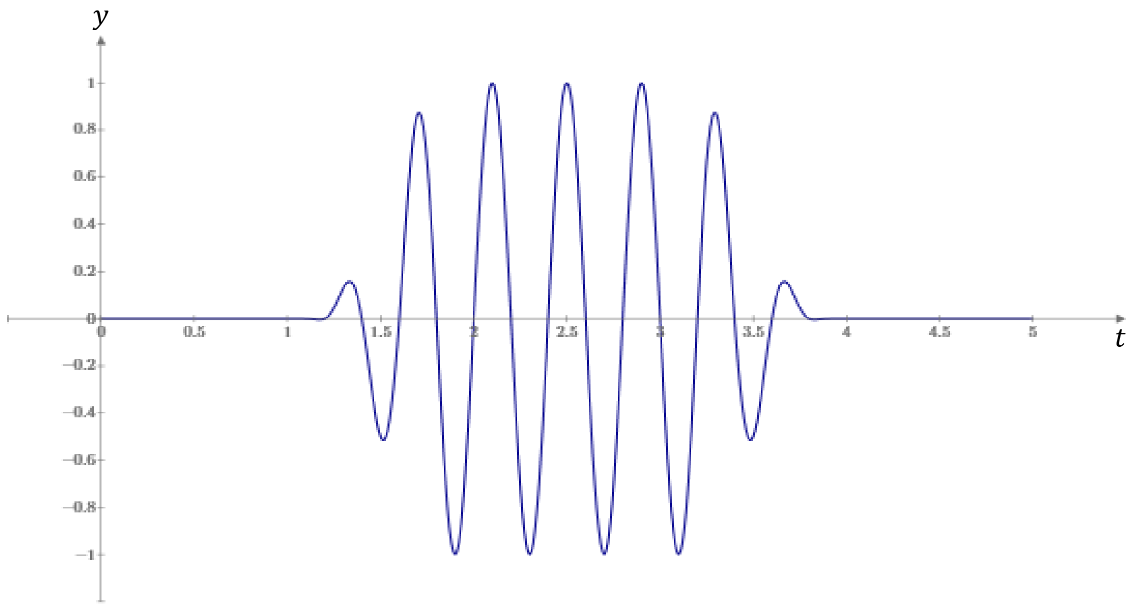

Figure 21. Harmonic function representing the earthquake motion.

Once the constitutive relations for both the spring force and the damping force, the electric field control function, and the external motion function have been defined, the solution to the problem can be developed.

Numerical Solution for Differential Algebraic Equations

The previously defined equations include: equation of motion (Eq. (11)), constitutive relations for the spring (Eq. (13)) and the dashpot (Eq. (22)), and external force (Eq. (26)). These can be used to define the following system of differential algebraic equations (DAE):

$$
\begin{gathered}
m \cdot \ddot{x}(t) = -F_s(t) - F_d(t) - m \cdot \ddot{y}(t) \\
F_s(t) = k \cdot x(t) \\
v(t) = \begin{cases} 0, & |F_d(t)| \le s_2 E, \\ \dfrac{F_d(t) - \operatorname{sgn}[F_d(t)]\, s_2 E}{2\sqrt{km}(\zeta_0 + s_1 E)}, & |F_d(t)| > s_2 E, \end{cases}
\end{gathered}
\tag{27}
$$

Now, given the fact that Eq. (27c) is not invertible, it is not possible to replace the values of $F_s(t)$ and $F_d(t)$ in the equation of motion; hence, there is a need to define the system (26) as a DAE system,

$$\ddot{x}(t) = \frac{1}{m}\left(-F_s(t) - F_d(t) - m \cdot \ddot{y}(t)\right)$$
$$F_s(t) = k \cdot x(t) \tag{28}$$
$$v(t) = \begin{cases} 0, & |F_d(t)| \leq s_2 E, \\ \dfrac{F_d(t) - \operatorname{sgn}[F_d(t)]\, s_2 E}{2\sqrt{km}(\zeta_0 + s_1 E)}, & |F_d(t)| > s_2 E, \end{cases}$$

Replacing $\dot{x}(t) = v(t)$ in Eq. (28a), and differentiating with respect to time Eq. (28b) renders

$$\dot{v}(t) = \frac{1}{m}\left(-F_s(t) - F_d(t) - m \cdot \ddot{y}(t)\right)$$
$$\dot{F}_s(t) = k \cdot v(t) \tag{29}$$
$$v(t) = \begin{cases} 0, & |F_d(t)| \leq s_2 E, \\ \dfrac{F_d(t) - \operatorname{sgn}[F_d(t)]\, s_2 E}{2\sqrt{km}(\zeta_0 + s_1 E)}, & |F_d(t)| > s_2 E, \end{cases}$$

Hence, in order to provide a solution for Eq. (29), the initial conditions are going to be $v(0) = v_0$, and $F_s(0) = F_{s_0}$, both directly related to the damping force F_d and the relative displacement response x, respectively.

For the DAE system (29), Darbha [3] shows non-existence of a classical solution predicated in the fact that Eq. (29) cannot be inverted and initial values cannot be given for the whole domain; he also shows that in the sense of Filippov a solution exists.

From such analysis a discretization of Eq. (29) is provided using the backward Euler scheme for a time t_{n+1}. Therefore, Eq. (29) can be written like:

$$
\begin{gathered}
\frac{1}{\Delta t}(v_{n+1} - v_n) = \frac{1}{m}\left(-F_{s_{n+1}} - F_{d_{n+1}} - m \cdot \ddot{y}_{n+1}\right) \\
\frac{1}{\Delta t}\left(F_{s_{n+1}} - F_{s_n}\right) = k \cdot v_{n+1} \\
v_{n+1} = \begin{cases} 0, & \left|F_{d_{n+1}}\right| \leq s_2 E_{n+1}, \\ \dfrac{F_{d_{n+1}} - \operatorname{sgn}\left[F_{d_{n+1}}\right] s_2 E_{n+1}}{2\sqrt{km}(\zeta_0 + s_1 E_{n+1})}, & \left|F_{d_{n+1}}\right| > s_2 E_{n+1}, \end{cases}
\end{gathered}
\qquad (30)
$$

Now, in Eq. (30b) $F_{s_{n+1}}$ can be written in terms of the variable v_{n+1}, and the result can be inserted into Eq. (30a). This renders the following:

$$
\begin{gathered}
\left[1 + \Delta t^2 \frac{k}{m}\right] v_{n+1} = \frac{\Delta t}{m}\left(\Gamma_{n+1} - F_{d_{n+1}}\right) \\
\Gamma_{n+1} = \frac{m}{\Delta t} v_n - F_{s_n} - m \cdot \ddot{y}_{n+1}
\end{gathered}
\qquad (31)
$$

where Γ_{n+1} is defined as the predictor, and it gathers the initial conditions of the system and all the known input information.

Then, from Eq. (31a) and Eq. (17) one can derive the following useful result:

$$
\operatorname{sgn}[\Gamma_{n+1}] = \operatorname{sgn}\left[F_{d_{n+1}}\right] \qquad (32)
$$

which is valid for both $\left|F_{d_{n+1}}\right| \leq s_2 E_{n+1}$, and $\left|F_{d_{n+1}}\right| > s_2 E_{n+1}$. This result in Eq. (32) can be effectively used in the solution process, given the fact that Γ_{n+1} is known, but $F_{d_{n+1}}$ is not.

Electric Field Discretization

Additionally, one issue arises from Eq. (30c) with respect to E_{n+1}. The electric field E at time t_{n+1} is a function of v_{n+1} which makes the system Eq. (30) implicit on v_{n+1}. One can use a more complicated scheme to solve the implicit set of equations, but that can be overcome by the analysis shown in Figure 22.

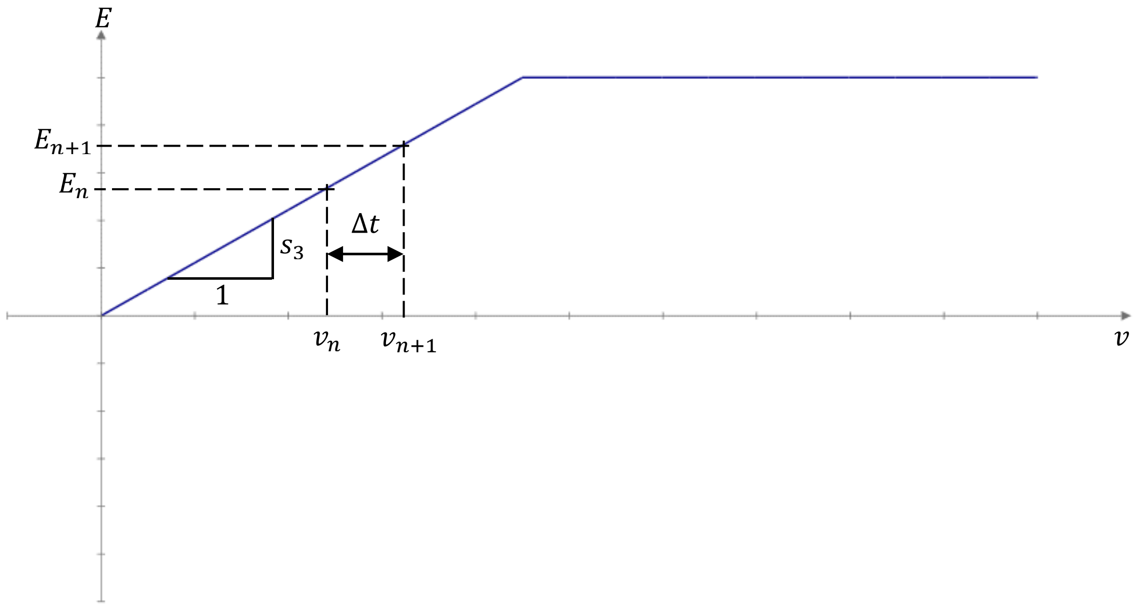

Figure 22. Discretization of the electric field control function.

From the figure above, the following relation is found,

$$E_{n+1} = E_n + s_3(v_{n+1} - v_n) \tag{33}$$

Now, it is obvious that as $\Delta t \rightarrow 0$, the second term on Eq. (33) will be very small compared to the first term, thus Eq. (33) can be seen as follows for a very small Δt,

$$\begin{gathered} \lim_{\Delta t \rightarrow 0} E_{n+1} = \lim_{\Delta t \rightarrow 0} E_n + \lim_{\Delta t \rightarrow 0} s_3(v_{n+1} - v_n) \\ E_{n+1} = E_n + 0 = s_3 v_n \end{gathered} \tag{34}$$

Therefore, the electrical field as a function of the relative velocity can be depicted as:

$$\begin{gathered} E_{n+1} = \begin{cases} s_3 v_n, & |v_n| \leq v_{lim}, \\ \operatorname{sgn}(v_n)\, E_{lim}, & |v_n| > v_{lim}, \end{cases} \\ s_3 = \frac{E_{lim}}{v_{lim}} \end{gathered} \tag{35}$$

where v_n is known.

Solution Algorithm Definition

Using all the previous definitions, the predictor-corrector algorithm proposed by Darbha [3] is applied to this work, as follows,

1: Input: k, m, Δt, s_1, s_2, s_3, v_n, F_{s_n}, $\ddot{y}_{n+1}$

2: Output: x_{n+1}, v_{n+1}, a_{n+1}, $F_{s_{n+1}}$, $F_{d_{n+1}}$

3: Calculate the electrical field:

4: **if** $|v_n| \leq v_{lim}$ **then**

5: $E_{n+1} = s_3 v_n$

6: **else**

7: $E_{n+1} = \mathrm{sgn}(v_n)\, E_{lim}$

8: **end if**

9: Predictor Step: Calculate the predictor $\Gamma_{n+1} = \frac{m}{\Delta t} v_n - F_{s_n} - m \cdot \ddot{y}_{n+1}$

10: Corrector step:

11: **if** $|\Gamma_{n+1}| \leq s_2 E_{n+1}$ **then**

12: $F_{d_{n+1}} = \Gamma_{n+1}$

$v_{n+1} = 0$

13: **else**

14:

$$F_{d_{n+1}} = \frac{\dfrac{\left(1 + \Delta t^2 \dfrac{k}{m}\right) s_2 E_{n+1}\, \mathrm{sgn}[\Gamma_{n+1}]}{2\sqrt{km}(\zeta_0 + s_1 E_{n+1})} + \dfrac{\Delta t}{m} \Gamma_{n+1}}{\dfrac{1 + \Delta t^2 \dfrac{k}{m}}{2\sqrt{km}(\zeta_0 + s_1 E_{n+1})} + \dfrac{\Delta t}{m}}$$

$$v_{n+1} = \frac{F_{d_{n+1}} - \mathrm{sgn}[\Gamma_{n+1}]\, s_2 E_{n+1}}{2\sqrt{km}(\zeta_0 + s_1 E_{n+1})}$$

15: **end if**

16: Calculate $F_{s_{n+1}}$, x_{n+1}, and a_{n+1}

$$F_{s_{n+1}} = F_{s_n} + \Delta t \cdot k \cdot v_{n+1}$$

$$x_{n+1} = \frac{1}{k} F_{s_{n+1}}$$

$$a_{n+1} = \frac{1}{m}\left(-F_{s_{n+1}} - F_{d_{n+1}} - m \cdot \ddot{y}_{n+1}\right)$$

The algorithm above can then be implemented using any software package. The next chapter of this work will analyze the results using data discussed in the first chapter in order to simulate a real life earthquake.

CHAPTER III

PARAMETER DEFINITION, RESULTS AND DISCUSSION, AND CONCLUSIONS

In this chapter, the information provided in the first chapter will be used to perform an analysis using the algorithm defined in the second chapter. Thus, several numerical and physical parameters are to be defined in order to find the response of three different analysis scenarios. The first scenario will be the response of the system when the electric field E is equal to zero. This scenario will show the common response of a Newtonian fluid on the dashpot. The second scenario will depict the response of the system for an electric field E that is different from zero. When the electric field is applied, an ER fluid starts to behave like a Bingham fluid, and a yield force has to be overcome to start of the motion. Finally, the third scenario will use the control function $E(v)$ which will help to provide a smooth increase of damping force as the velocity increases.

Parameter Definition

In order to pick the values to run a study, some of the values reviewed in the first chapter are used. Hence, the following data were chosen for natural frequency,

$$
\begin{aligned}
m_s &= 40kg \\
k &= 2\frac{kgf}{mm} \\
\omega_n &= \sqrt{\frac{k}{m_s}} = 22.143\frac{rad}{s} \\
f_n &= \frac{\omega_n}{2\pi} = 3.524Hz
\end{aligned}
\tag{36}
$$

The natural frequency, according to Figure 1, is close to that of a three story building.

Then, the definition of the electro-rheological damping parameters renders

$$\begin{gathered}\zeta_0 = 5\% \\ \zeta_{1,max} = 5\% \\ F_{0,max} = 6kgf \\ E_{f,max} = 3\frac{kV}{mm} \\ v_{lim} = 45\frac{mm}{s} \\ E_{lim} = 3\frac{kV}{mm}\end{gathered} \tag{37}$$

From (37) and using Eq. (22), and (23), the coefficients s_1, s_2, and s_3 can be calculated.

Next, to define the forcing function, four cases of study are proposed. The next four subsections define those cases.

Case 1:

This case analyzes the most common event: a high frequency earthquake with small amplitude (Figures 23 and 24), as defined in Chapter I. The analysis frame is set at 5 seconds.

$$\begin{gathered}y(t) = y_0 \cdot h\left(t - \frac{t_f}{2}\right) \cdot \sin(\omega t) \\ y_0 = 1cm \\ \omega = 5\pi\frac{rad}{s} \\ f = \frac{\omega}{2\pi} = 2.5Hz\end{gathered} \tag{38}$$

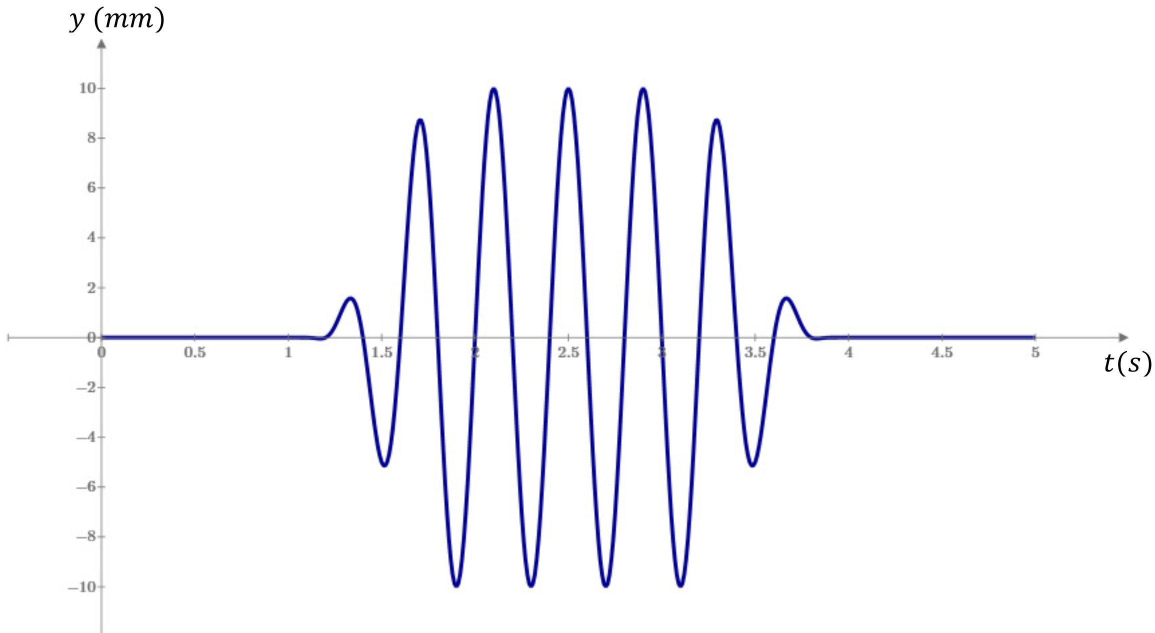

Figure 23. Earthquake sine-bump forcing function for Case 1. Amplitude is 1 cm, and frequency is 2,5 Hz.

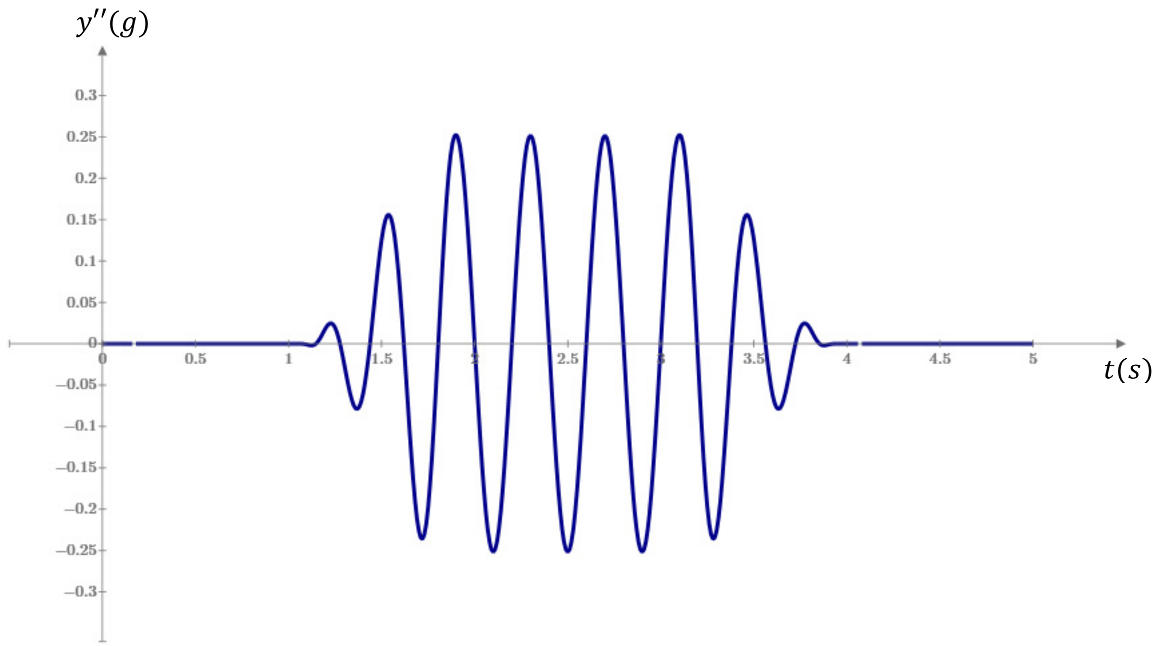

Figure 24. Earthquake acceleration for Case 1. Amplitude is 0.25 g.

Case 2:

Case 2 is a low frequency earthquake with higher amplitude (Figures 25 and 26), as defined in Chapter I. The analysis frame is again 5 seconds.

$$
\begin{gathered}
y(t) = y_0 \cdot h\left(t - \frac{t_f}{2}\right) \cdot \sin(\omega t) \\
y_0 = 10cm \\
\omega = 2\pi \frac{rad}{s} \\
f = \frac{\omega}{2\pi} = 1Hz
\end{gathered}
\tag{39}
$$

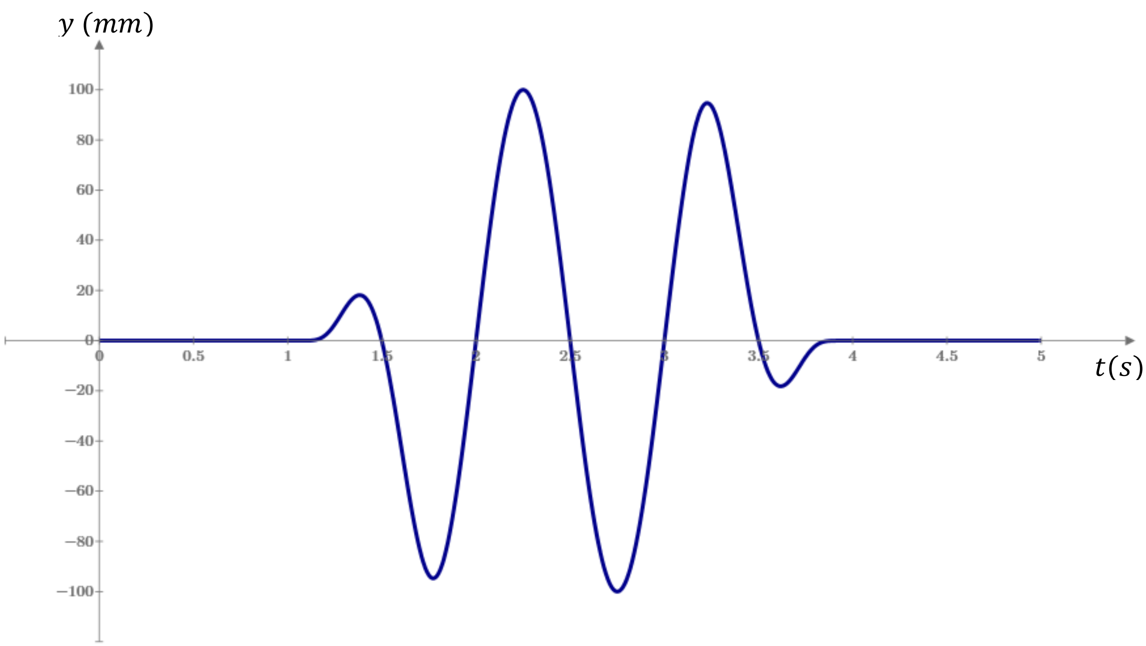

Figure 25. Earthquake sine-bump forcing function for Case 2. Amplitude is 10 cm, and frequency is 1 Hz.

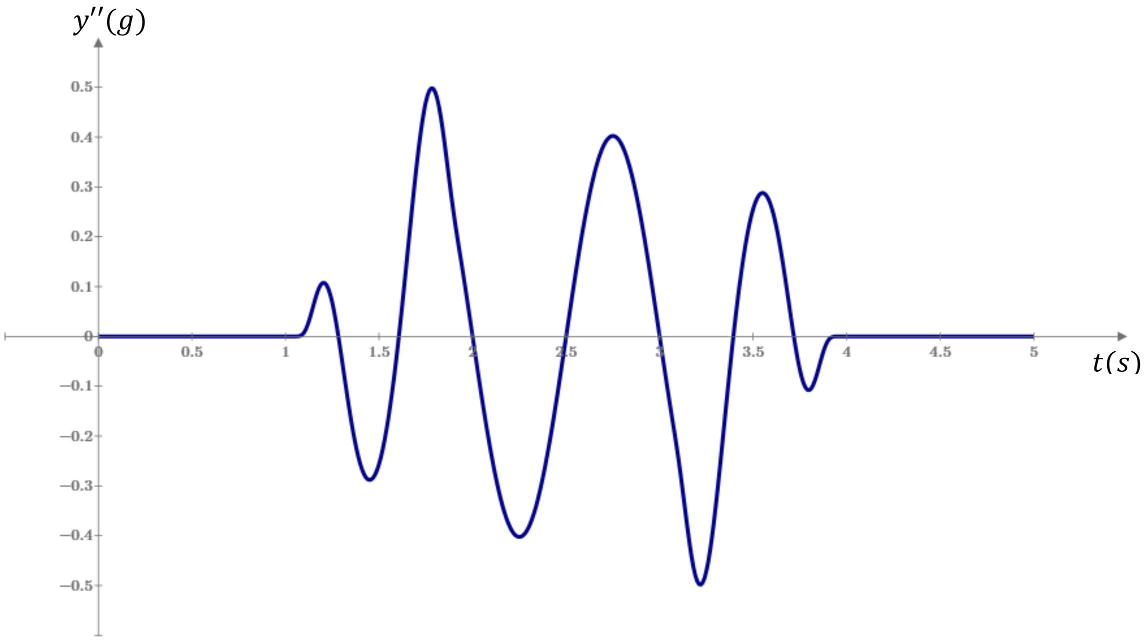

Figure 26. Earthquake acceleration for Case 2. Amplitude is 0.5 g.

Case 3:

The third case is a high frequency sine forcing function with low amplitude (Figures 27 and 28). The analysis frame is 5 seconds.

$$y(t) = y_0 \cdot \sin(\omega t)$$
$$y_0 = 1cm$$
$$\omega = 5\pi \frac{rad}{s} \quad (40)$$
$$f = \frac{\omega}{2\pi} = 2.5Hz$$

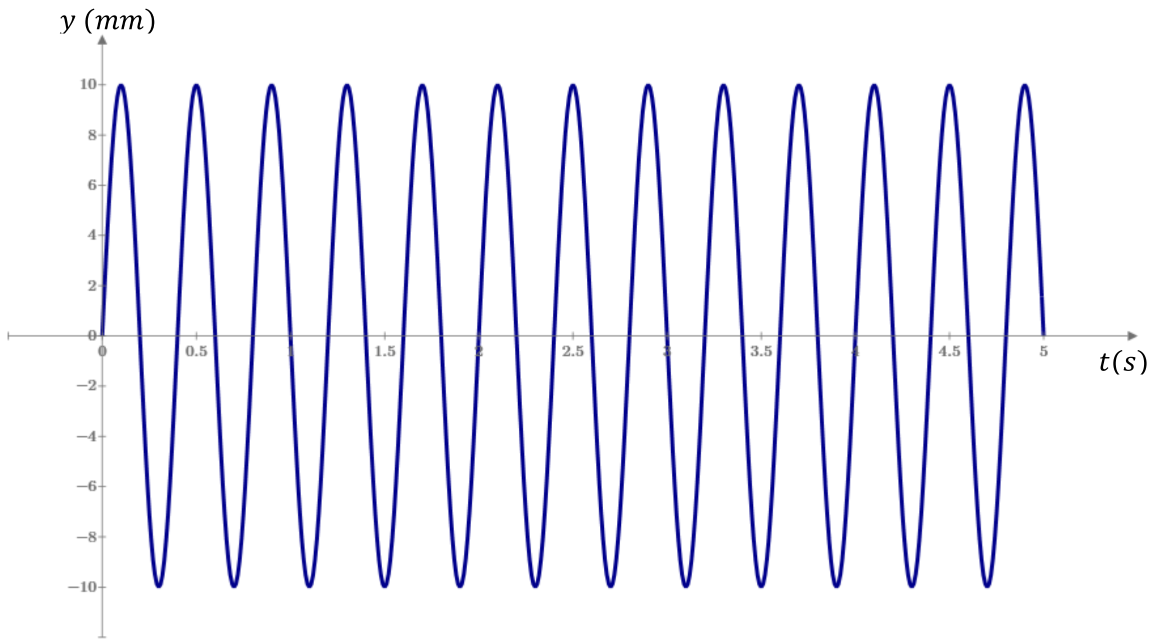

Figure 27. Sine forcing function for Case 3. Amplitude is 1cm, and frequency is 2,5 Hz.

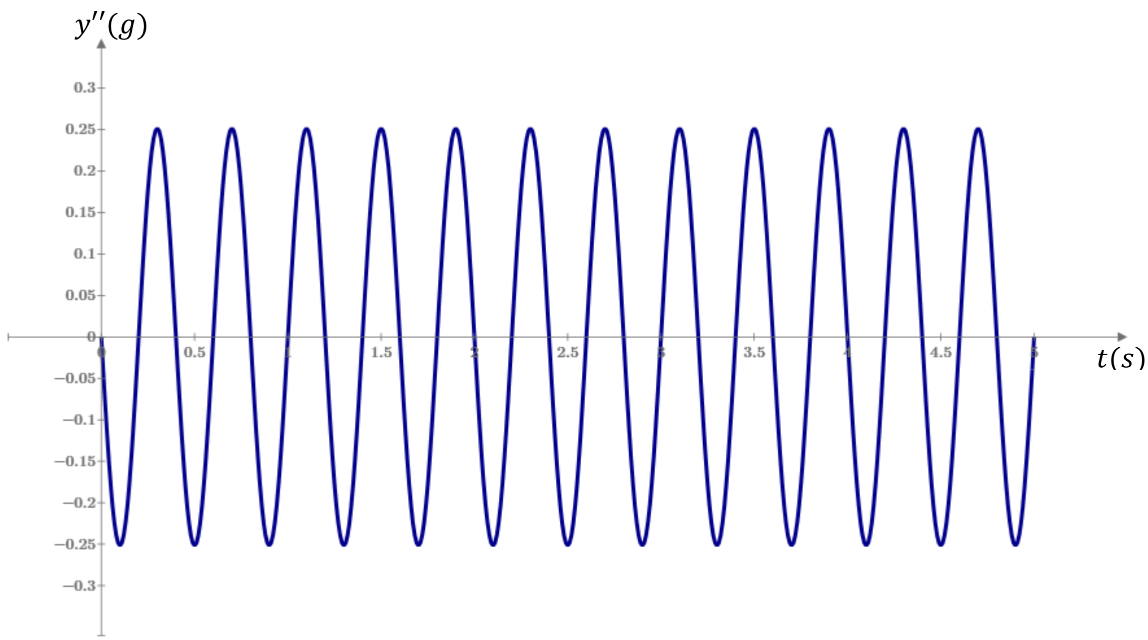

Figure 28. Acceleration for Case 3. Amplitude is 0.25 g.

Case 4:

Case 4 is a low frequency sine forcing function with higher amplitude (Figures 29 and 30). The analysis frame is 5 seconds.

$$y(t) = y_0 \cdot \sin(\omega t)$$
$$y_0 = 10cm$$
$$\omega = 2\pi \frac{rad}{s} \tag{41}$$
$$f = \frac{\omega}{2\pi} = 1Hz$$

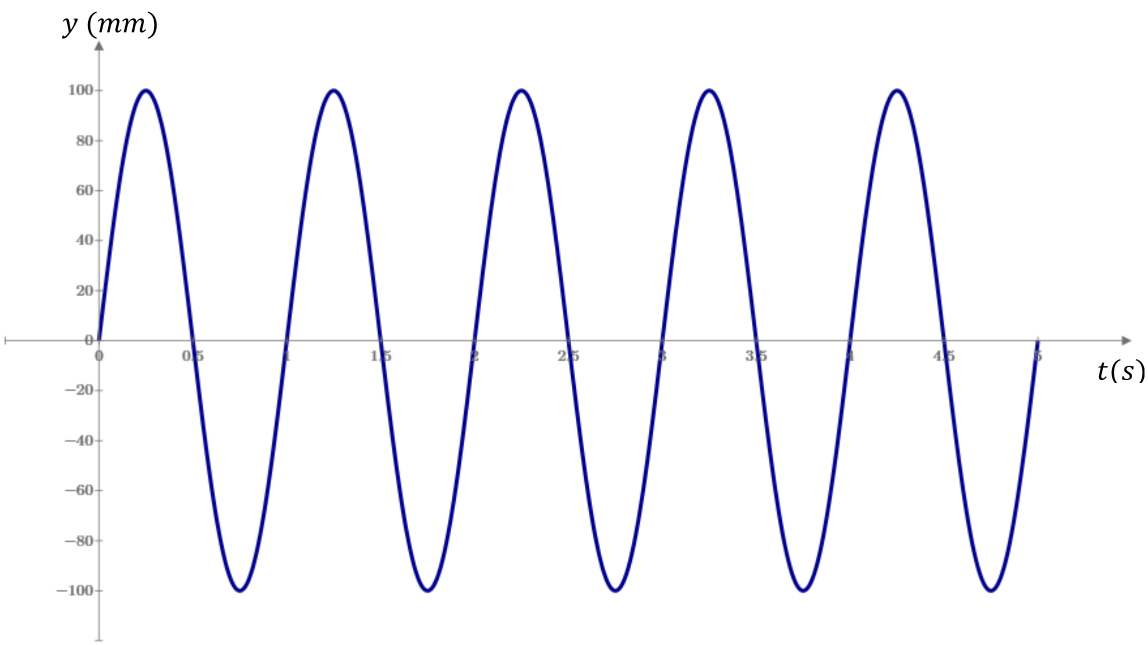

Figure 29. Earthquake sine-bump forcing function for Case 4. Amplitude is 10 cm, and frequency is 1 Hz.

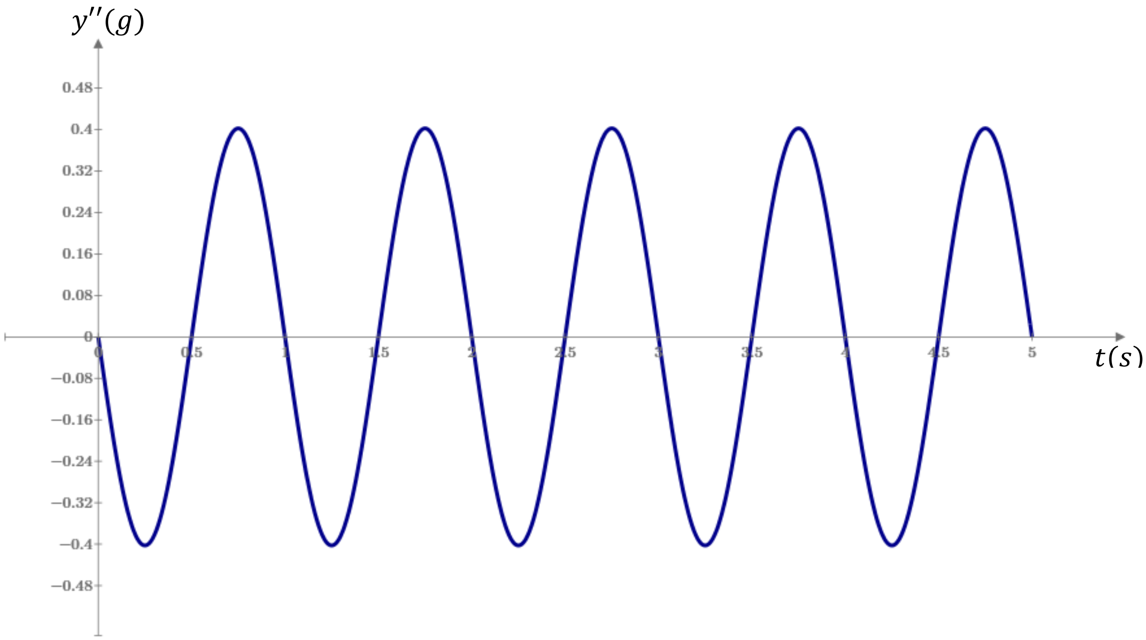

Figure 30. Earthquake acceleration for Case 4. Amplitude is 0.4 g.

Thus, the analysis will be programmed and run to find the response for the 4 case scenarios.

Results and Discussion

In order to get the system response for the 4 cases of study we just defined, we will use the proposed algorithm from Chapter II, the input parameters defined above, and a programming software (Figure 31).

The provided initial conditions are $v_0 = 0\frac{mm}{s}$, $F_{s_0} = 0kgf$, that make sense for a state before an earthquake.

The complimentary information needed for the study is defined as follows:

$$n = 5000,\ t_f = 5s,\ t_0 = 0s$$
$$\Delta t = \frac{t_f - t_0}{n} = 0.001 \tag{42}$$

$$
\begin{array}{l}
t_0 \leftarrow t_0 \\
v_0 \leftarrow v_0 \\
F_{s_0} \leftarrow F_{s0} \\
E_{f_0} \leftarrow 0 \\
\text{for } i \in 0, 1 .. n-1 \\
\quad \left\| \begin{array}{l}
t_{i+1} \leftarrow t_i + \Delta t \\
\Gamma_{i+1} \leftarrow \dfrac{m_s}{\Delta t} \cdot v_i - m_s \cdot y''\left(t_{i+1}\right) - F_{s_i} \\
\text{if } \left|v_i\right| \le v_{lim} \\
\quad \left\| E_{f_{i+1}} \leftarrow s_3 \cdot v_i \right. \\
\text{else} \\
\quad \left\| E_{f_{i+1}} \leftarrow \operatorname{sign}\left(v_i\right) \cdot E_{lim} \right. \\
\text{if } \left|\Gamma_{i+1}\right| \le s_2 \cdot \left|E_{f_{i+1}}\right| \\
\quad \left\| \begin{array}{l}
F_{d_{i+1}} \leftarrow \Gamma_{i+1} \\
v_{i+1} \leftarrow 0 \ \dfrac{m}{s}
\end{array} \right. \\
\text{else} \\
\quad \left\| \begin{array}{l}
F_{d_{i+1}} \leftarrow \dfrac{\dfrac{\left(1 + \Delta t^2 \cdot \dfrac{k}{m_s}\right) \cdot s_2 \cdot \left|E_{f_{i+1}}\right| \cdot \operatorname{sign}\left(\Gamma_{i+1}\right)}{2 \cdot \sqrt{k \cdot m_s} \cdot \left(\zeta_0 + s_1 \cdot \left|E_{f_{i+1}}\right|\right)} + \dfrac{\Delta t}{m_s} \cdot \Gamma_{i+1}}{\dfrac{1 + \Delta t^2 \cdot \dfrac{k}{m_s}}{2 \cdot \sqrt{k \cdot m_s} \cdot \left(\zeta_0 + s_1 \cdot \left|E_{f_{i+1}}\right|\right)} + \dfrac{\Delta t}{m_s}} \\
v_{i+1} \leftarrow \dfrac{F_{d_{i+1}} - \operatorname{sign}\left(\Gamma_{i+1}\right) \cdot s_2 \cdot \left|E_{f_{i+1}}\right|}{2 \cdot \sqrt{k \cdot m_s} \cdot \left(\zeta_0 + s_1 \cdot \left|E_{f_{i+1}}\right|\right)}
\end{array} \right. \\
F_{s_{i+1}} \leftarrow F_{s_i} + \Delta t \cdot k \cdot v_{i+1} \\
x_{i+1} \leftarrow \dfrac{1}{k} \cdot F_{s_{i+1}} \\
a_{i+1} \leftarrow \dfrac{1}{m_s} \cdot \left(-F_{s_{i+1}} - F_{d_{i+1}} - m_s \cdot y''\left(t_{i+1}\right)\right)
\end{array} \right. \\
P \leftarrow \operatorname{augment}\left(t, x, v, a, F_s, F_d\right)
\end{array}
$$

Figure 31. Algorithm implementation.

Displacement Response

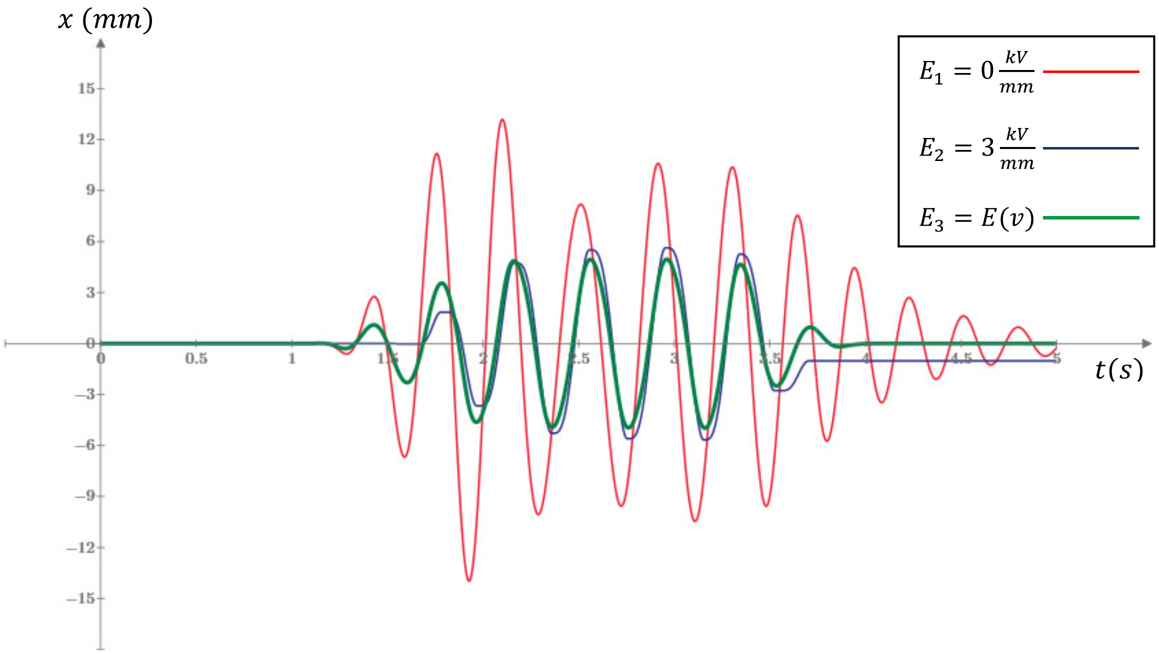

Figure 32. Case 1: Displacement response.

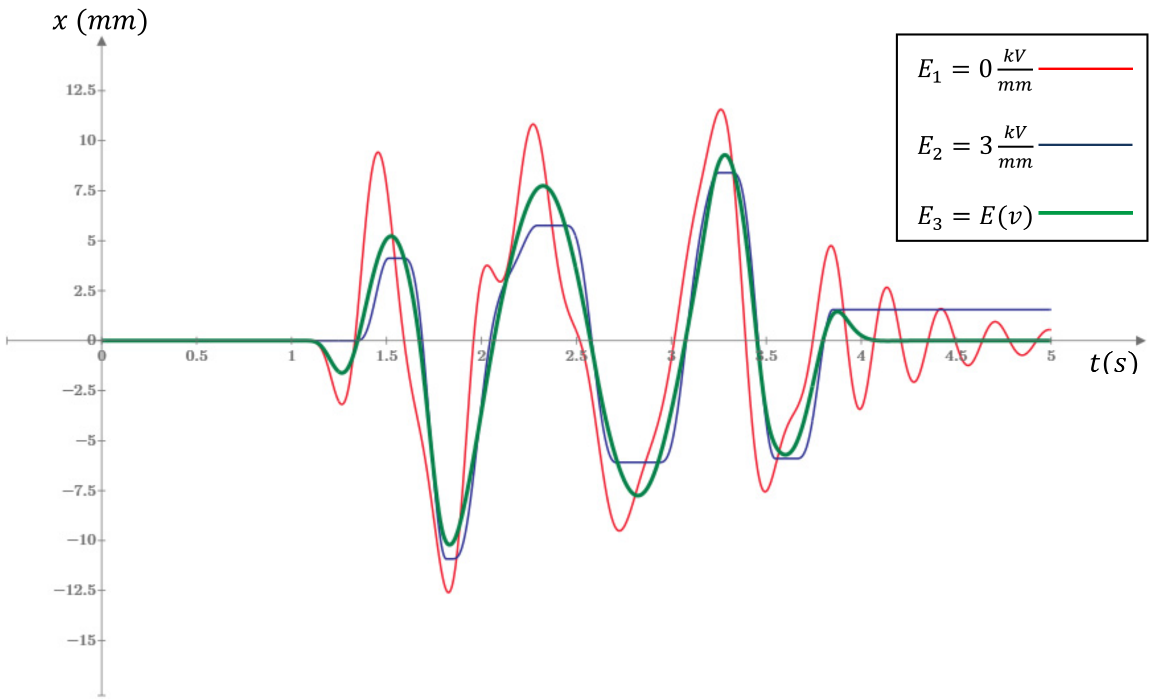

Figure 33. Case 2: Displacement response.

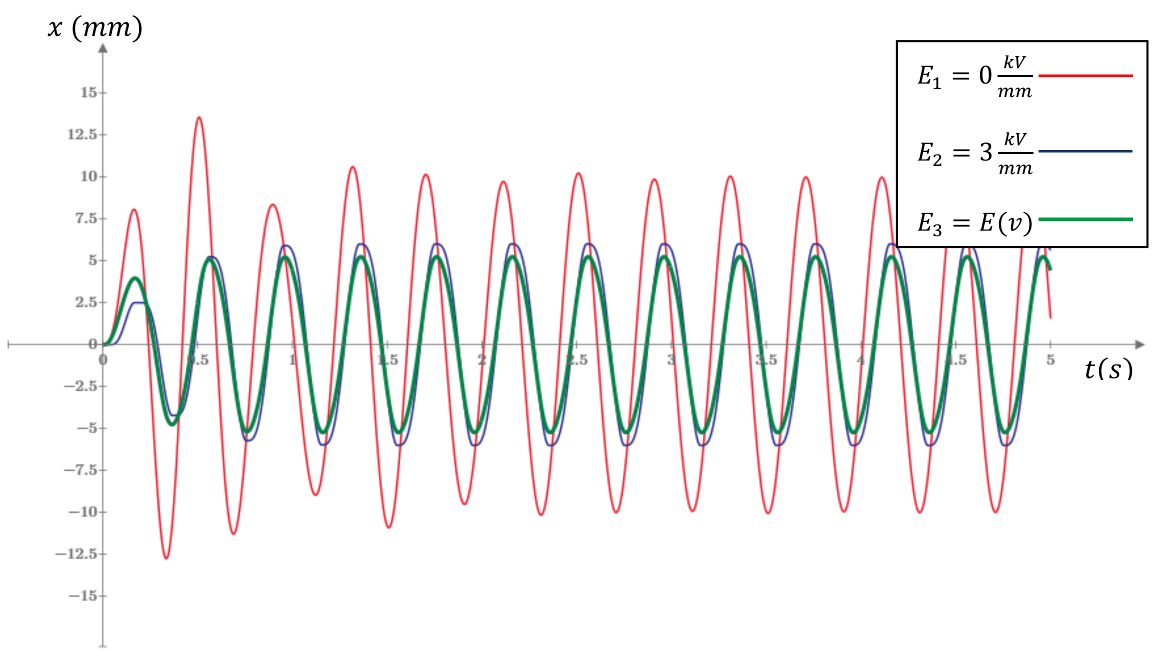

Figure 34. Case 3: Displacement response.

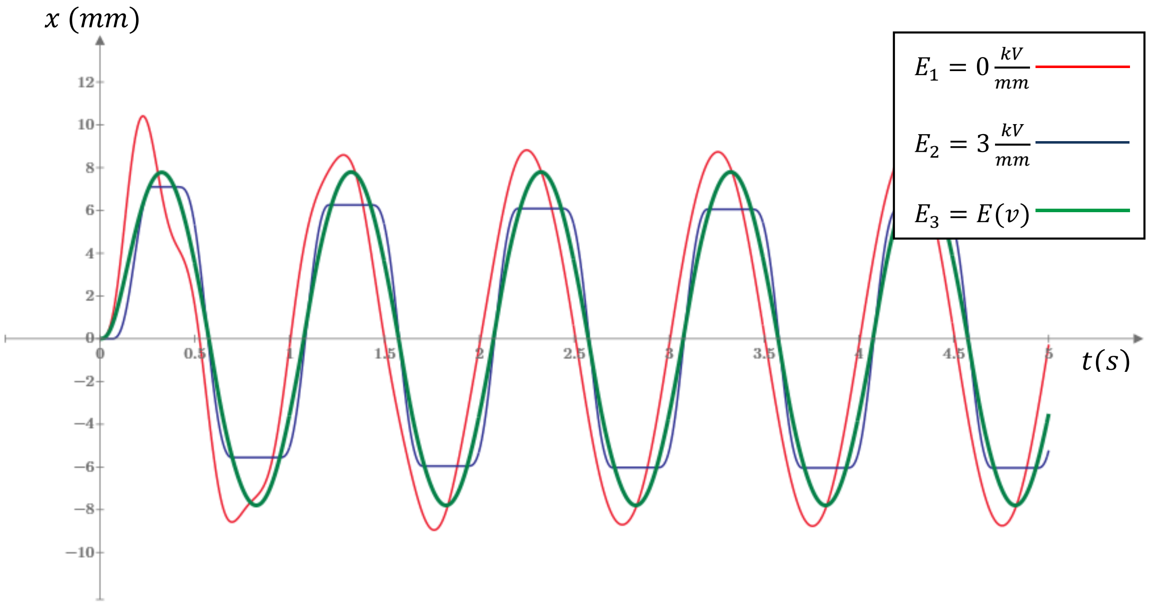

Figure 35. Case 4: Displacement response.

For Cases 1 and 2 (Figures 32 and 33), it is evident that for the control function $E_3 = E(v)$ in Eq. (23), the response is smooth and the time frame is shorter. Also, for both higher and lower frequency earthquakes, the maximum displacement amplitude is smaller as well.

For Cases 3 and 4 (Figures 34 and 35), once the response achieves steady state, the amplitude for the control function is shorter and smoother.

Take into account that when $E \neq 0$, the frequency response is out of phase with respect to the forcing function frequency.

Velocity Response

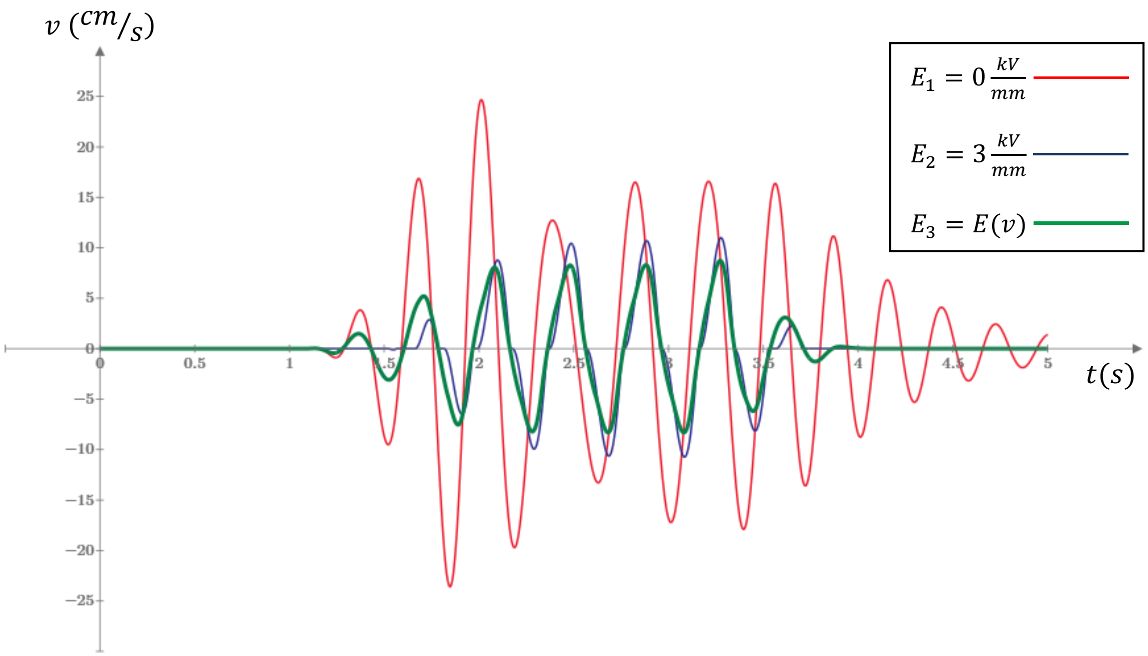

Figure 36. Case 1: Velocity response.

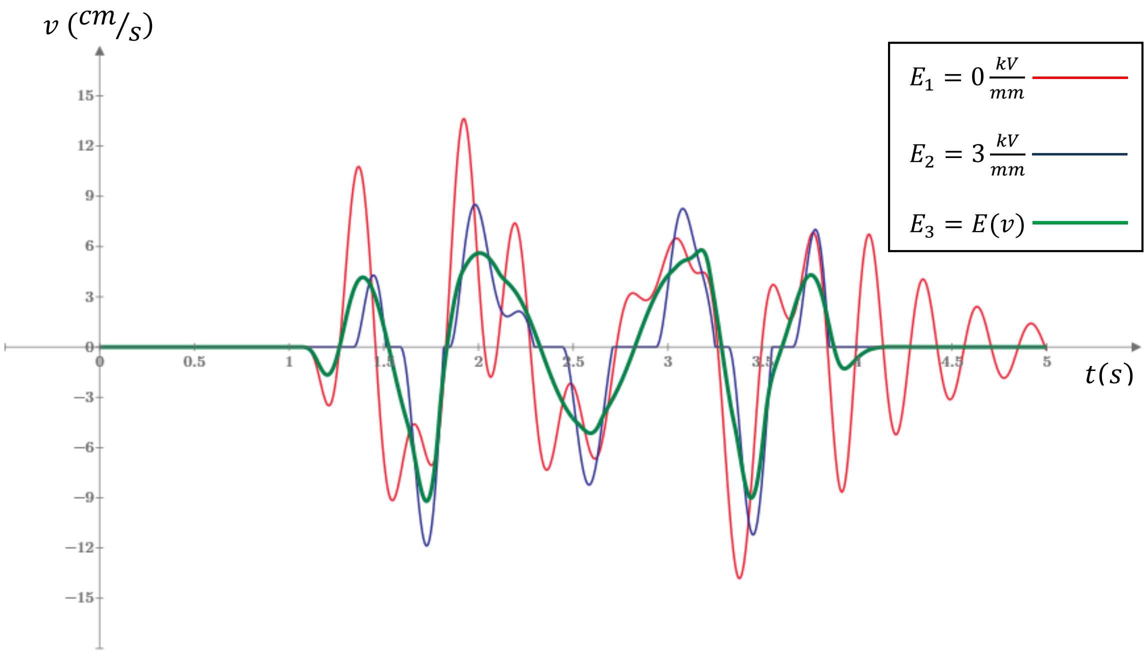

Figure 37. Case 2: Velocity response.

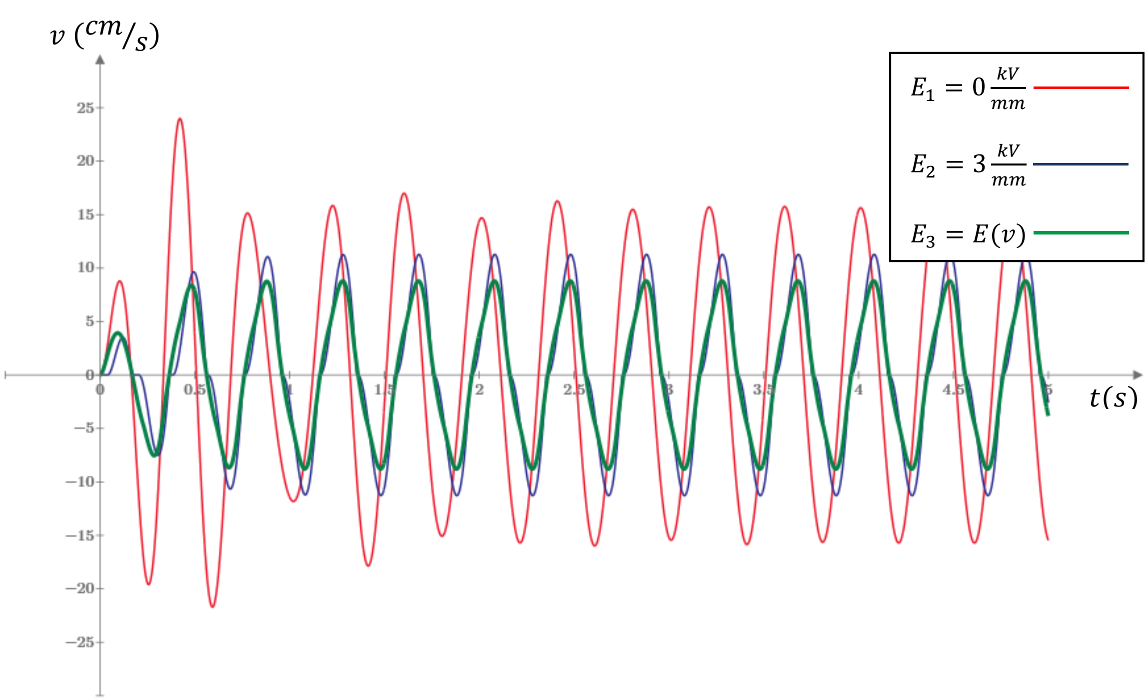

Figure 38. Case 3: Velocity response.

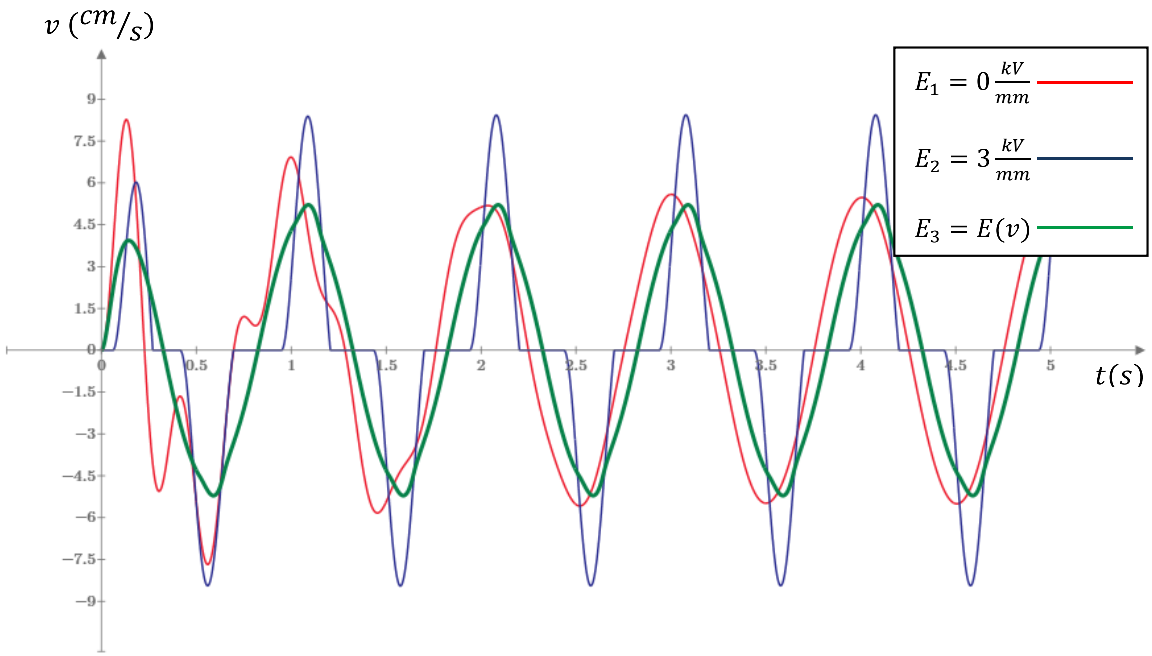

Figure 39. Case 4: Velocity response.

For electric fields set at either zero or maximum, for all the cases of study (Figures from 36 to 39) there are higher peaks and irregular response. However, when the control function is used, the response becomes regular with smooth transitions.

Acceleration Response

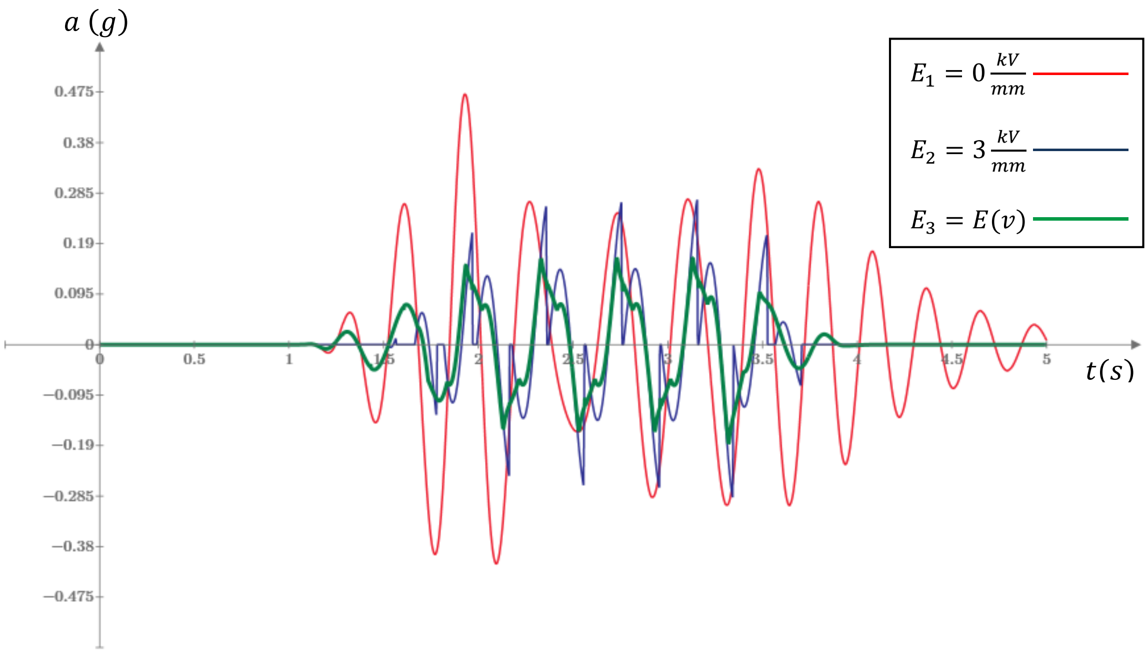

Figure 40. Case 1: Acceleration response.

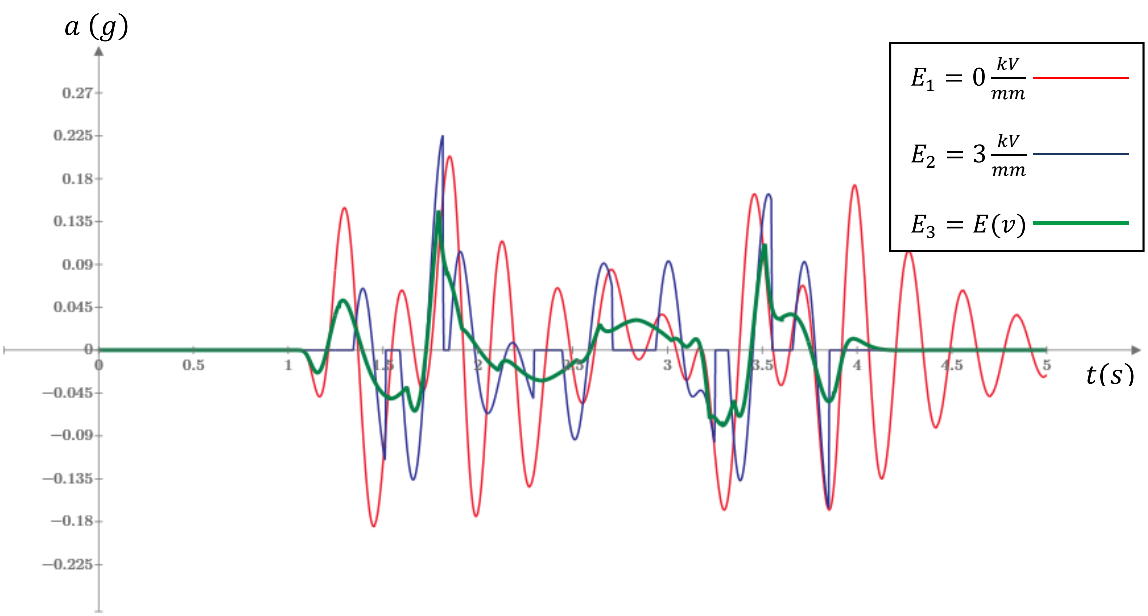

Figure 41. Case 2: Acceleration response.

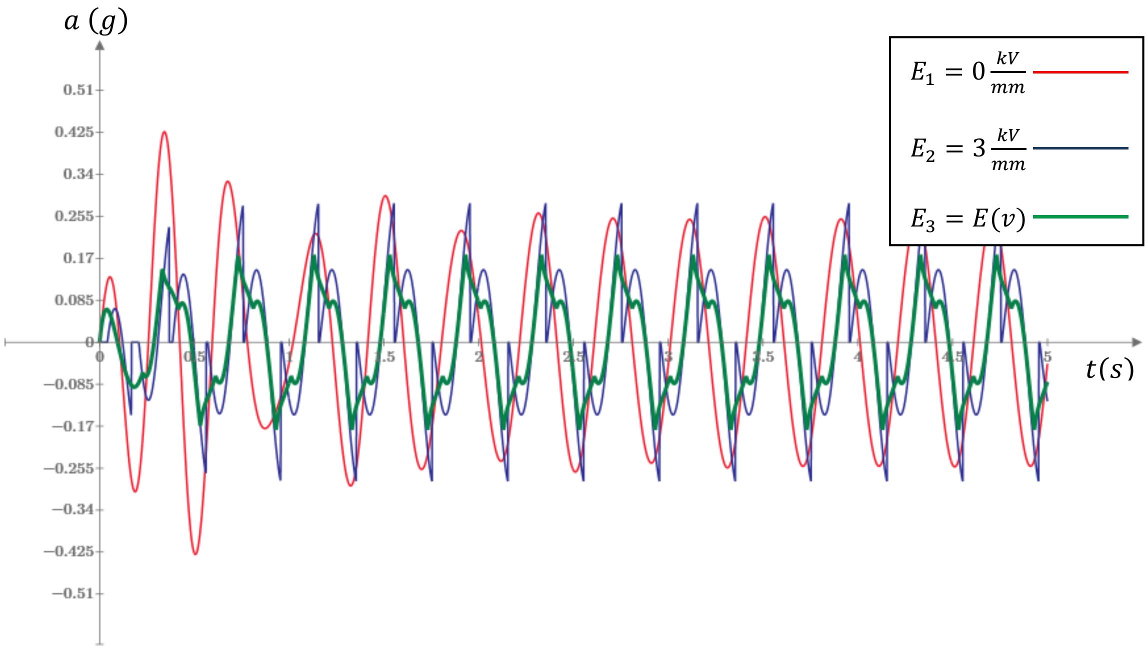

Figure 42. Case 3: Acceleration response.

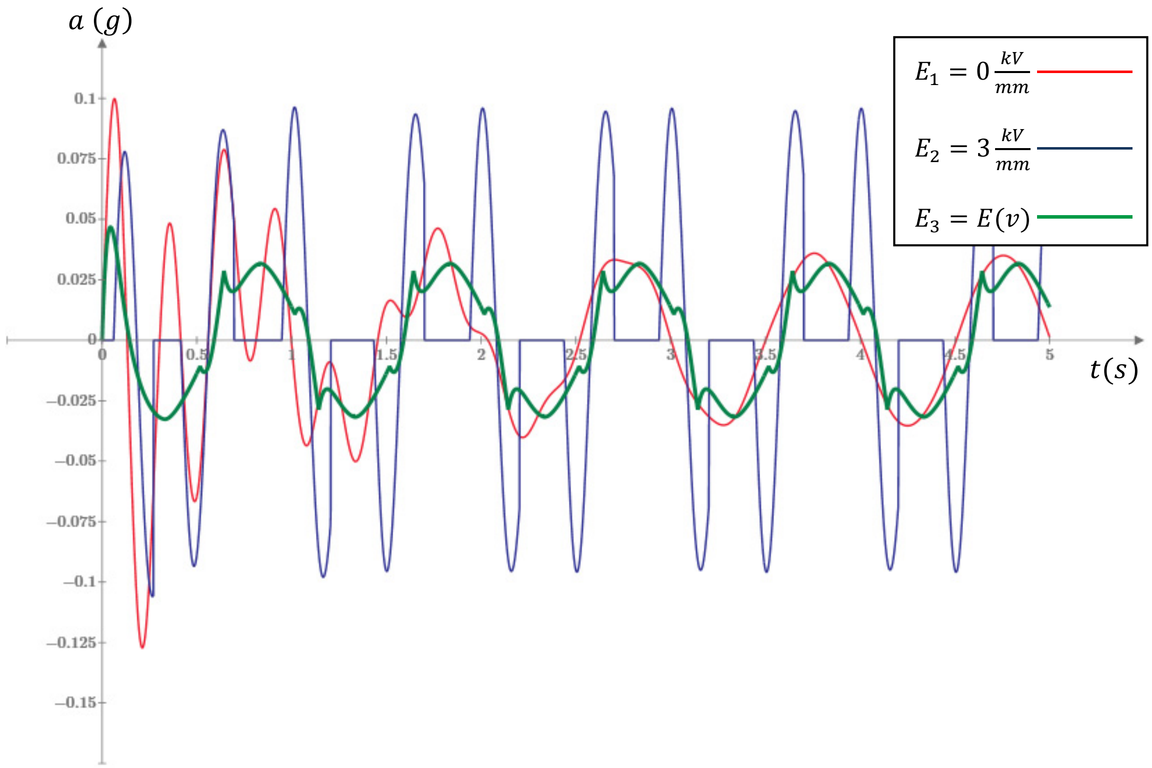

Figure 43. Case 4: Acceleration response.

The most important result in this study is the acceleration. As stated in Chapter I, the acceleration response is responsible for the aftermath in a seismic event. The control function method was proven to decrease the acceleration amplitude considerably (Figures from 40 to 43), even in the worst case scenario, Case 1.

Spring Force Response

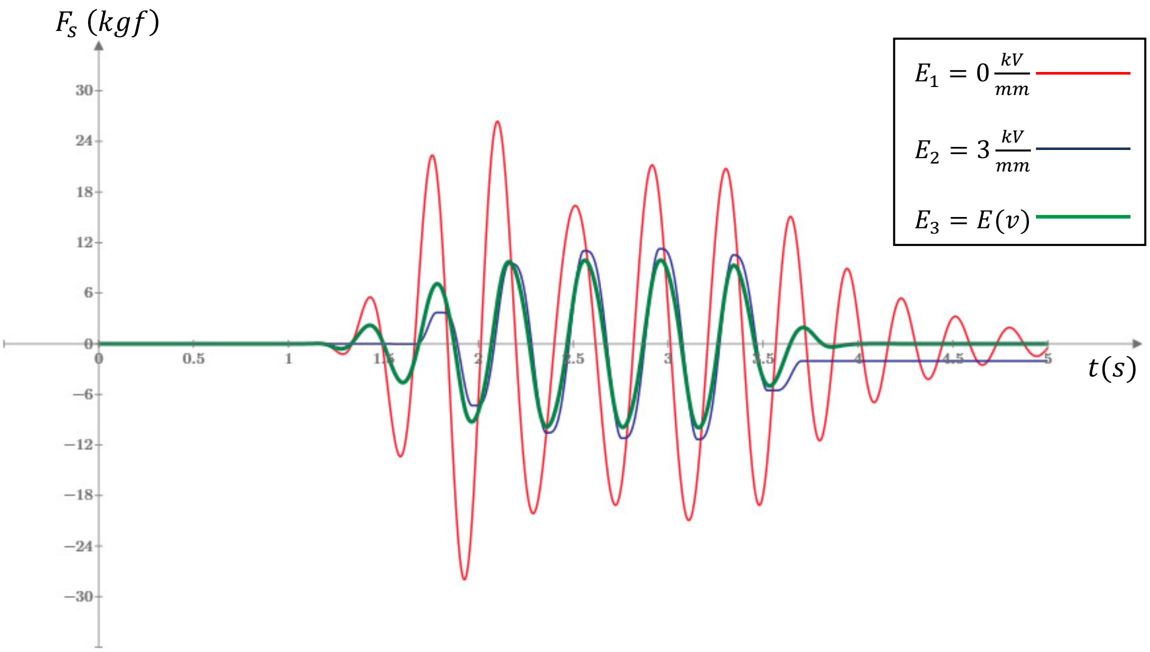

Figure 44. Case 1: Spring force response.

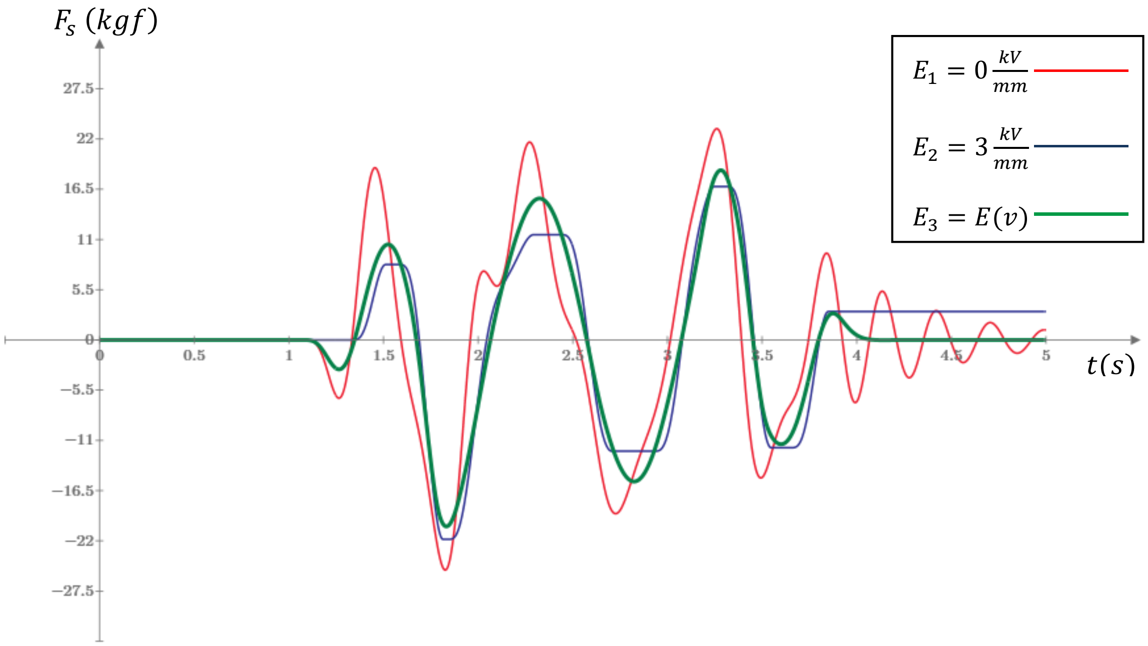

Figure 45. Case 2: Spring force response.

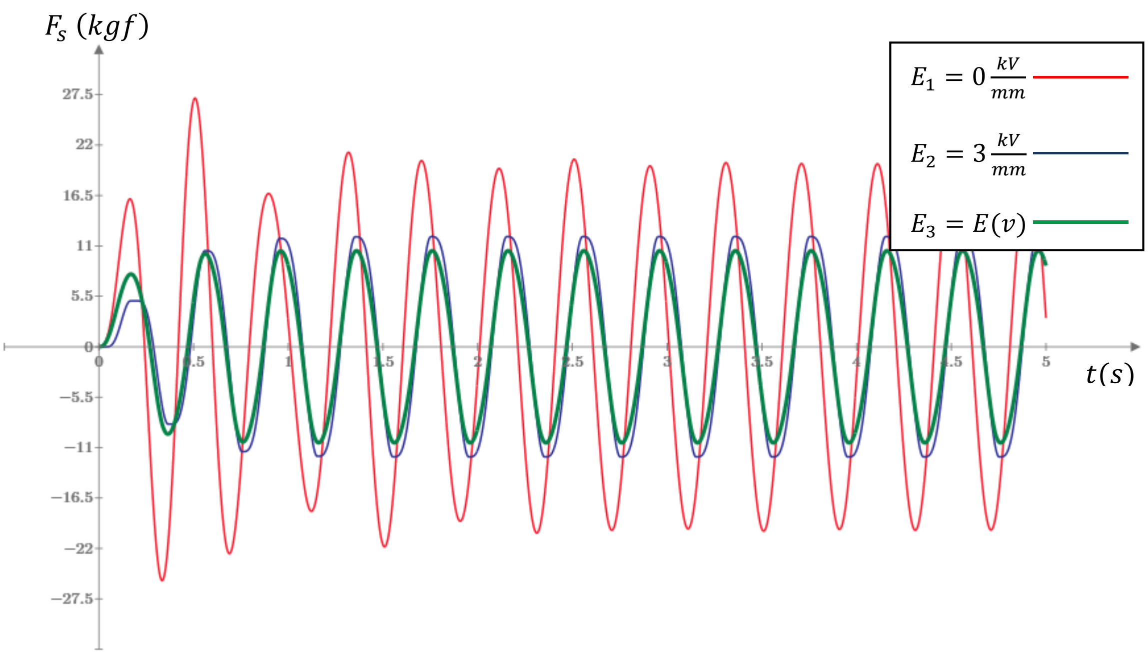

Figure 46. Case 3: Spring force response.

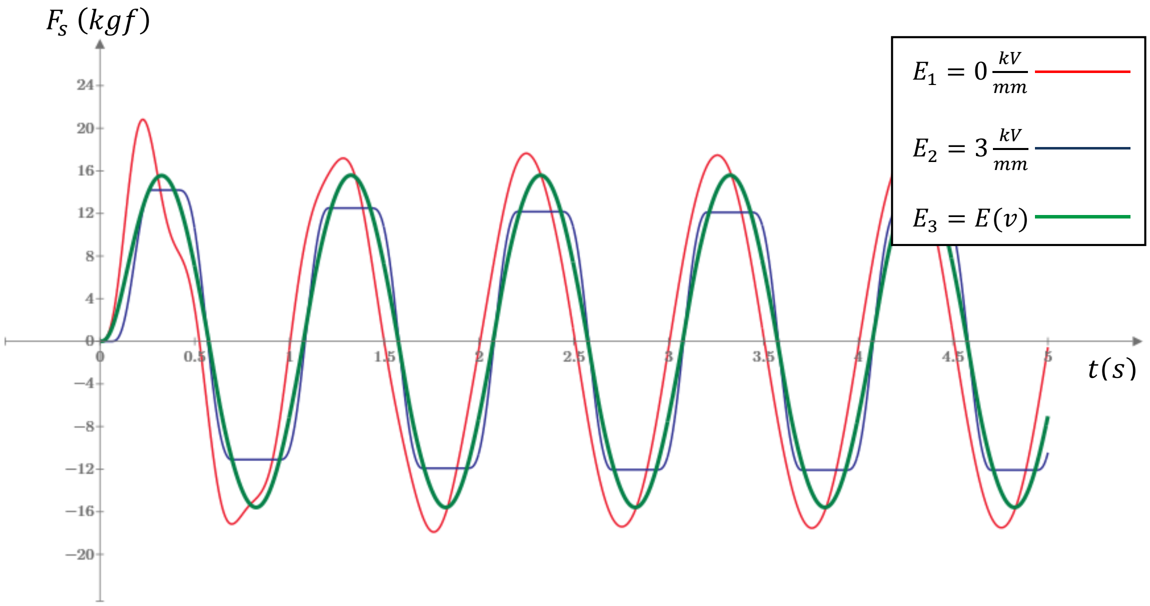

Figure 47. Case 4: Spring force response.

The spring response (Figures from 44 to 47) is directly and linearly related to the displacement; thus the same analysis applies to it.

Damping Force Response

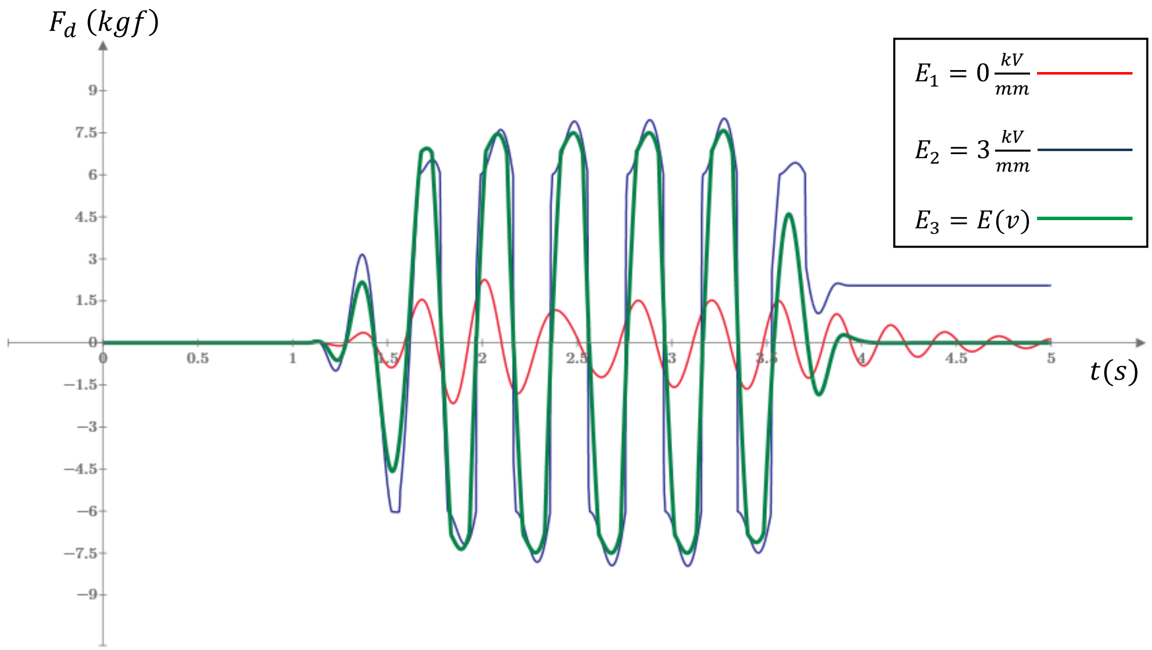

Figure 48. Case 1: Damping force response.

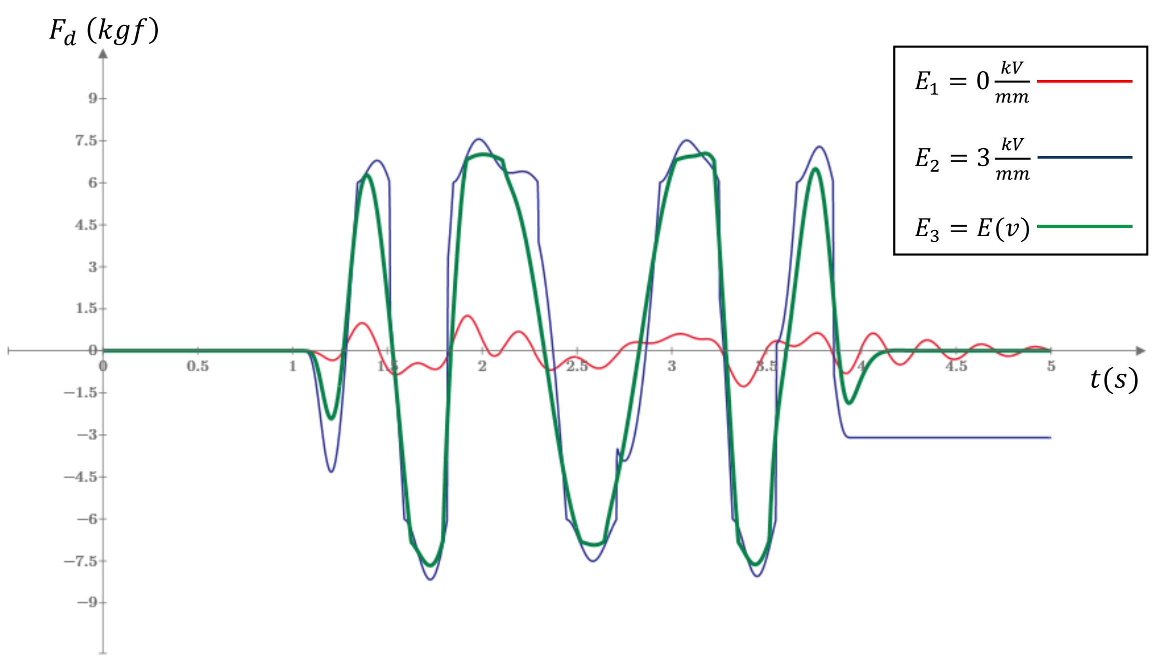

Figure 49. Case 2: Damping force response.

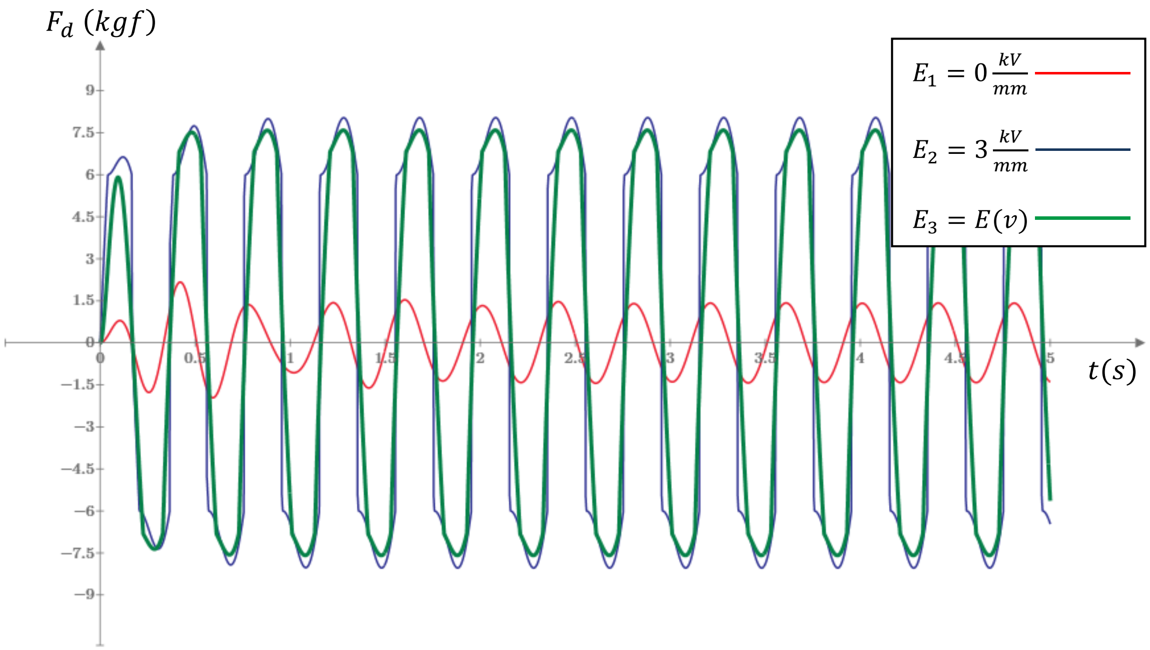

Figure 50. Case 3: Damping force response.

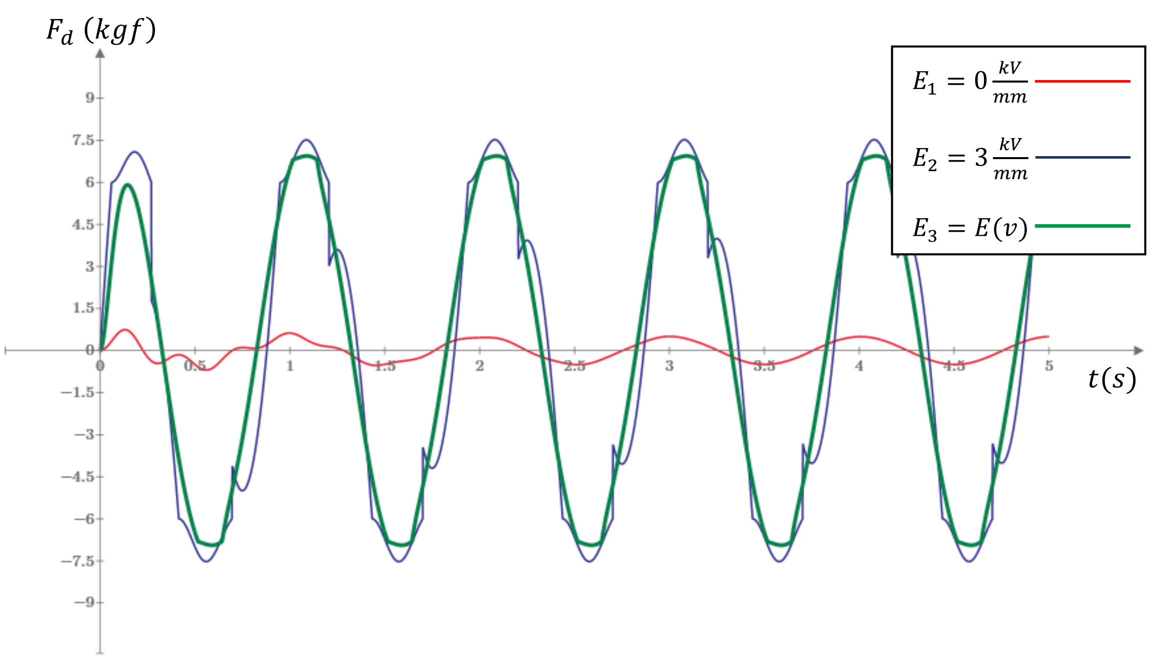

Figure 51. Case 4: Damping force response.

Another important factor in this analysis is the damping force response. It is interesting to see that even though the damping force amplitude is very similar in the case of maximum electric field and control function, the displacement and acceleration amplitude for the control function is considerably smaller (Figures from 48 to 51).

Damping Force vs. Velocity

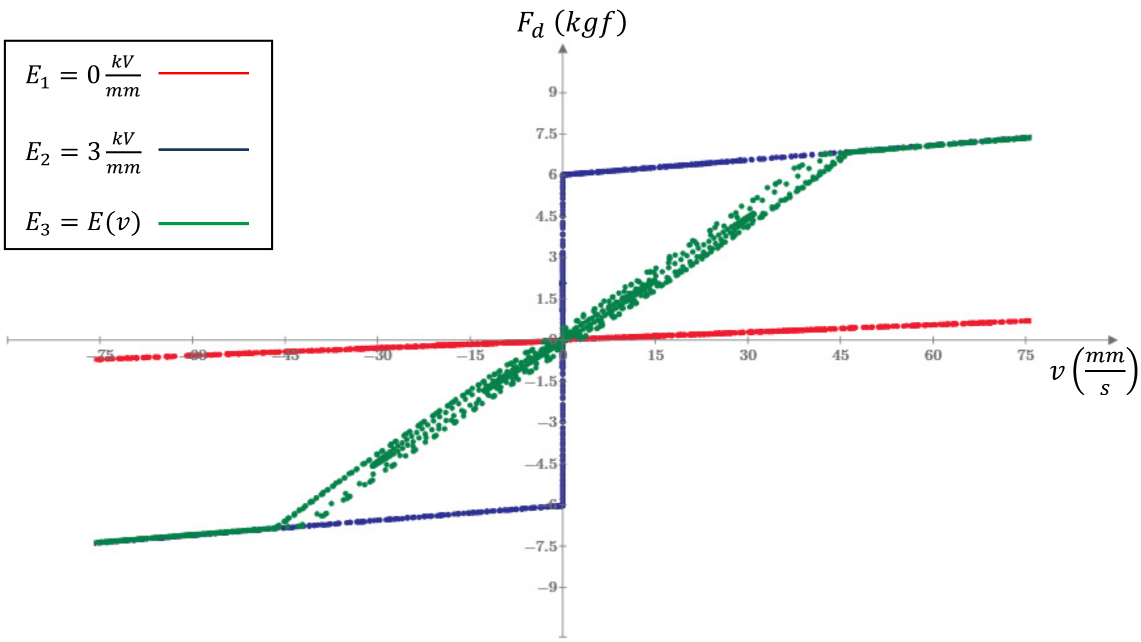

Figure 52. Case 1: Damping force vs velocity.

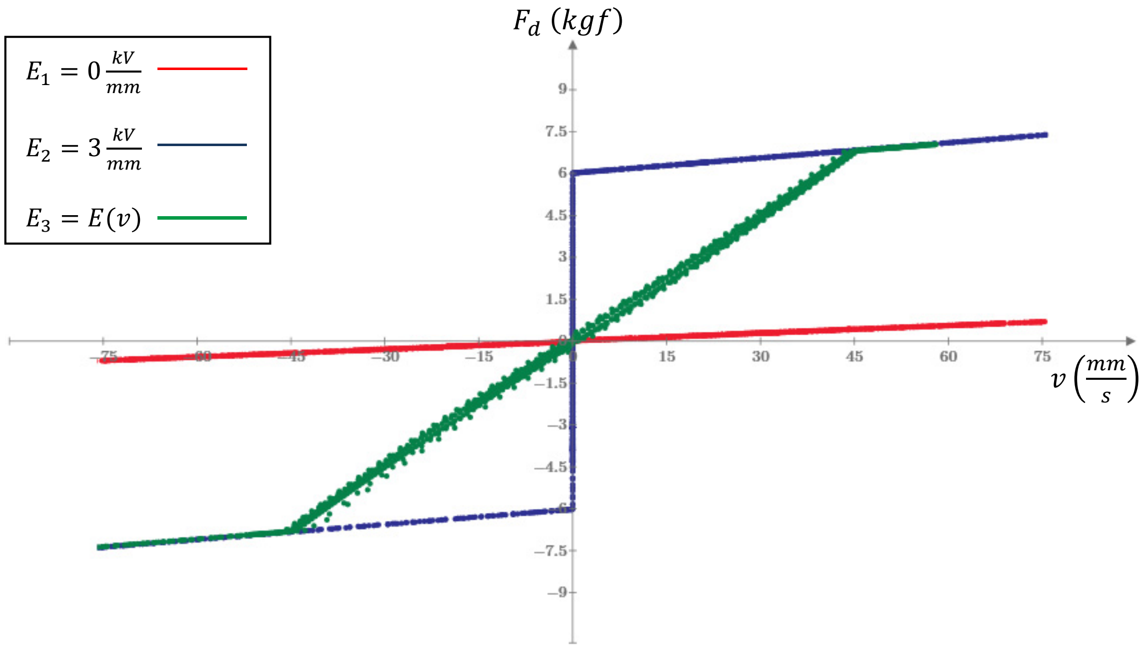

Figure 53. Case 2: Damping force vs velocity.

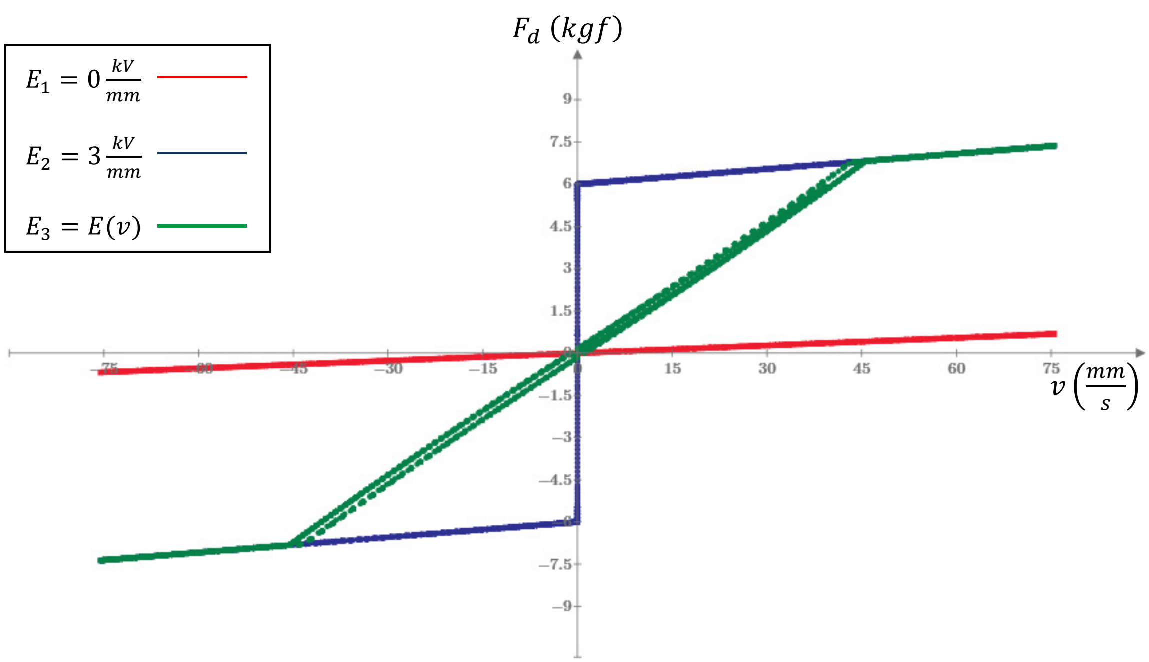

Figure 54. Case 3: Damping force vs velocity.

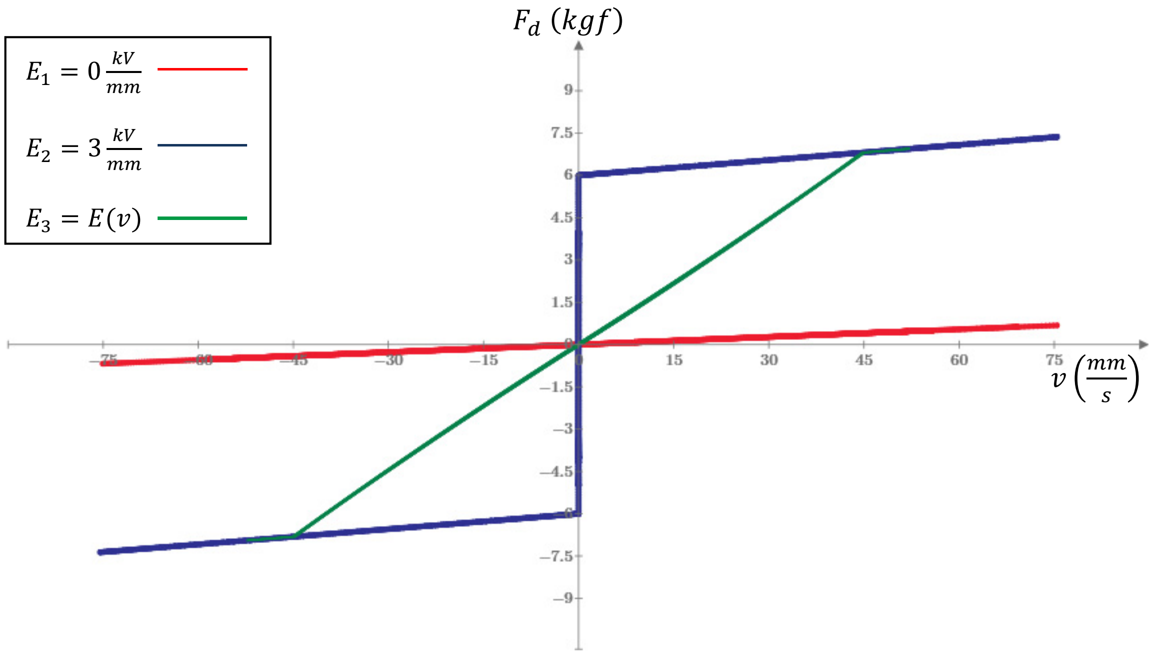

Figure 55. Case 4: Damping force vs velocity.

From the Figures 52 to 55, we can see that the control function creates a sort of Newtonian fluid zone with higher slope, which means higher viscosity. This zone of higher viscosity provides the control for the amplitude response for both displacement and acceleration.

Conclusions

When the constitutive equations in a lumped parameter system cannot be defined as functions of the respective kinematical variables, there is still the chance that the phenomenon can be expressed as a differential algebraic equation system where the kinematical variables are functions of the forces, and a numerical solution can be found.

One has to choose consistent initial conditions in order to find a solution. For instance, for a seismic event, the initial velocity and initial displacement must be zero given the physical initial state of the system.

The use of a control function that provides electric field as a function of the velocity of the system is proven to give smoothness and stability to a vibrating system. Further analysis into the type of control function can lead to even better control results over the system response.

A study with not only a variable damping but also with a variable stiffness can be of great interest in the field of controlling seismic effects.

The application of electric field to an ER fluid, increasing the damping, is not enough to ensure a smaller acceleration amplitude response. Those peaks where the acceleration increases suddenly can be a dangerous effect over any main or secondary structure experiencing an earthquake. An electric field controlling function proved to manage the changes in acceleration effectively.

REFERENCES

[1] C. Arnold, Earthquake Effects on Buildings, Federal Emergency Management Agency FEMA, Chapter IV, 2013, http://www.fema.gov/media-library-data/20130726-1556-20490-0102/fema454_chapter4.pdf

[2] K. R. Rajagopal, A. S. Wineman, Flow of electro-rheological materials, Acta Mech. 91, 57-75 (1992), Springer-Verlag 1992.

[3] S. Darbha, K. B. Nakshatrala, K. R. Rajagopal, On the vibrations of lumped parameter systems governed by differential-algebraic equations, arXiv:0911.0137 [cs.NA] [v5] Tue, 20 Jul 2010.